Stochastic Models in Probability and Statistics
Vol. 2, No. 2, 2025, pp. 211–219. Research Paper

SMPS

Neutrosophic ZLindley distribution with applications

Lazhar Benkhelifa*

Department of Mathematics, Mohamed Khider University, Biskra, Algeria

Email(s): lazhar.benkhelifa@univ-biskra.dz

Abstract. This paper introduces the neutrosophic ZLindley distribution, extending the classical Lindley distribution within a neutrosophic framework to more effectively handle datasets characterized by uncertainty, imprecision, and incompleteness. We derive several properties of the proposed model and estimate its neutrosophic parameter using the method of maximum likelihood. A simulation study is performed to evaluate the estimator's performance. Finally, the advantages of the proposed model are demonstrated through applications to real datasets.

Keywords: Data analysis, Imprecise data, Maximum likelihood method, Neutrosophic random variable, ZLindley distribution.

1 Introduction

When data or portions of it are uncertain to varying degrees, the neutrosophic theory presents a powerful framework to handle uncertainty. Although certainty is highlighted as a basic requirement in classical mathematics, real-world data usually has high levels of fuzziness, uncertainty, and incompleteness, often surpassing the existence of fully determinate information. To fill this gap, Smarandache (2014) suggested the notion of neutrosophic statistics as a logical extension of classical statistics. The purpose of this expansion is to manage imprecise or uncertain data and their accompanying statistical probability distributions by describing them via sets of values that approximate the original idealized precise values. In these situations, the usefulness of neutrosophic statistical techniques is especially clear, since they allow us to analyze and arrange neutrosophic data in ways that more accurately and nuancedly reveal underlying patterns than are possible with traditional statistical techniques.

In recent years, researchers have developed neutrosophic analogues of numerous classical probability distributions. For example, Alhabib et al. (2018) introduced neutrosophic versions of the Poisson, exponential, and uniform distributions. Alhasan and Smarandache (2019) formulated the neutrosophic Weibull distribution. Khan Sherwani el al. (2021) introduced the neutrosophic beta distribution. Alduais and Khan (2025) developed the neutrosophic extensions of the log-normal distribution. Bashir el al. (2024) presented a modified neutrosophic log-logistic distribution. Bashir el al. (2025) proposed the neutrosophic Lindley distribution. Alduais and Khan (2025) explored the neutrosophic gamma distribution. Razmkhah el al. (2026) investigated the neutrosophic Birnbaum-Saunders distribution.

*Corresponding author
Received: 25 September 2025 / Accepted: 11 December 2025
DOI: 10.22067/smps.2025.95633.1053

 https://smps.um.ac.ir

Some existing methods for interval data involve calculating classical statistical measures on the lower and upper bounds of the data intervals. This separate calculation and combination of bounds can neglect the internal details and variability within the interval, resulting in a loss of important information. For this reason, Razmkhah el al. (2026) propose a new statistical method for handling interval data that directly uses the entire data interval. This approach allows for the preservation of the complete information contained within the data interval, leading to richer datasets for analysis and offering more accurate and reliable statistical results.

In this paper, we propose the neutrosophic ZLindley distribution (NZLD). The rest of the paper is organized as follows: Section 2 introduces the NZLD. Some statistical properties are presented in Section 3. Section 4 describes the estimation procedure for the neutrosophic parameter. A simulation study is carried out in Section 5. In Section 6, real datasets are analyzed to demonstrate the applicability of the proposed distribution. Finally, a brief conclusions are provided in Section 7.

2 Neutrosophic ZLindley distribution

The increasing complexity of real-life data demands more flexible and robust probability models. Traditional parametric models, like exponential, Weibull, and normal distributions, often fail to capture the nuanced behavior of modern survival or reliability data. To address these challenges, researchers and practitioners have recently proposed several models. Saaidia et al. (2024) introduced the ZLindley and studied some of its properties. A random variable X follows the ZLindley distribution (ZLD) if its cumulative distribution function (cdf) is

$$F(x,\theta) = 1 - \left(1 + \frac{\theta x}{2(\theta+1)}\right) e^{-\theta x}, \quad x > 0,$$

where $\theta > 0$ is the scale parameter. The corresponding probability density function (pdf) of X is given by

$$f(x,\theta) = \frac{\theta(1+2\theta+\theta x)}{2(\theta+1)}\ e^{-\theta x}, \quad x > 0.$$

A neutrosophic random variable $X_N = [X_L, X_U]$ follows a neutrosophic ZLD (NZLD) with neutrosophic scale parameter $\theta_N \in [\theta_L, \theta_U]$, if its neutrosophic pdf (npdf) is

$$f_N(x_N,\theta_N) = \frac{\theta_N(1+2\theta_N+\theta_N x_N)}{2(\theta_N+1)}\ e^{-\theta_N x_N}, \tag{1}$$

where X_L and X_U (or θ_L and θ_U) represent the lower bound and upper bound of X_N (or θ_N). $f_N(x_N,\theta_N)$ given in (1) defines a valid neutrosophic pdf on $(0,+\infty)$, since

$$\begin{aligned}
\int_0^{+\infty} f_N(x_N;\theta_N)dx_N &= \int_0^{+\infty} \frac{\theta_N(1+2\theta_N+\theta_N x_N)}{2(\theta_N+1)}\ e^{-\theta_N x_N} dx_N \\
&= \frac{\theta_N}{2(\theta_N+1)}\left(\int_0^{+\infty}(1+2\theta_N)\ e^{-\theta_N x_N} dx_N + \int_0^{+\infty} \theta_N x_N\ e^{-\theta_N x_N} dx_N\right) \\
&= \frac{\theta_N}{2(\theta_N+1)}\left(\frac{2\theta_N+1}{\theta_N} + \frac{1}{\theta_N}\right) \\
&= 1.
\end{aligned}$$

The corresponding neutrosophic survival function of $f_N(x_N,\theta_N)$ is given by

$$S_N(x_N,\theta_N) = 1 - F_N(x_N,\theta_N) = \left(1 + \frac{\theta_N x_N}{2(\theta_N+1)}\right) e^{-\theta_N x_N}, \quad x_N > 0. \tag{2}$$

while the neutrosophic hazard function (NHF) of the NZLD is

$$h_N(x_N;\theta_N) = \frac{(1+2\theta_N+\theta_N x_N)\theta_N}{2+2\theta_N+\theta_N x_N}, \quad x_N > 0. \tag{3}$$

The plot of the npdf of the NZLD is shown in Figure 1, while the plots of the ncdf and NHF of the NZLD are presented in Figures 2 and 3, respectively. In Figure 1, $\theta_N \in [0.5,1.0],[0.25,0.40],[0.55,2.0]$ and $[2,3.5]$ from left panel to right. Figure 2, $\theta_N \in [0.5,1.0],[0.5,2.0],[1.5,3.0]$ and $[0.1,0.35]$ from left panel to right. Figure 3, $\theta_N \in [0.05,0.1],[1.5,2.0],[0.35,0.45]$ and $[2.25,3.5]$ from left panel to right. In all figures, the black curves represent the function values at the lower and upper bounds of the neutrosophic scale parameter θ_N. The colored regions illustrate how the function values vary as the parameter shifts across its indeterminacy interval.

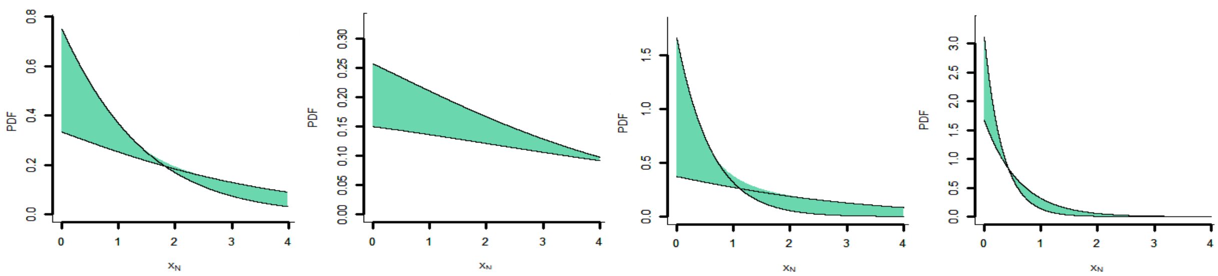

Figure 1: The plot of npdf of the NZLD in (1).

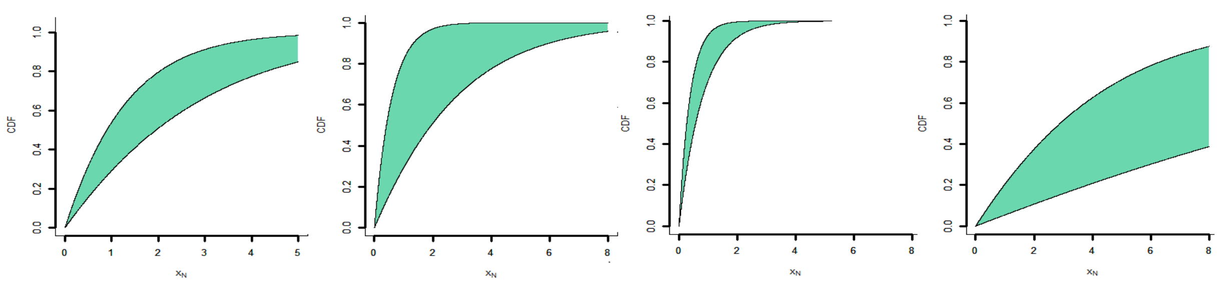

Figure 2: The plot of $F_N(x_N,\theta_N)$ in (2).

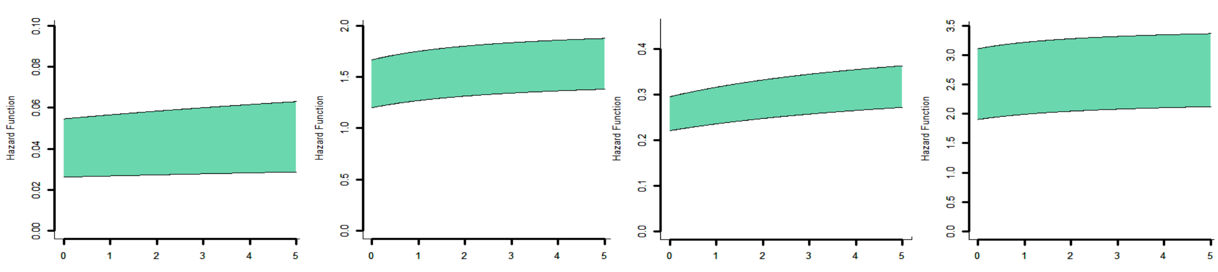

Figure 3: the plot of NHF of the NZLD in (3).

3 Statistical properties of the NZLD

Suppose X has npdf $f_N(x_N,\theta_N)$ as given in (1). Then, it is easy to show that the r-th raw moment of X_N is

$$\mu_r' = E(X_N^r;\theta_N) = \frac{(r+2+2\theta_N)\Gamma(r+1)}{2\theta_N^r(1+\theta_N))},$$

where $\Gamma(.)$ is the complete gamma function. Hence, we the first four raw moments are

$$\mu_1' = \frac{(3+2\theta_N)}{2\theta_N(1+\theta_N)}, \mu_2' = \frac{2(2+\theta_N)}{\theta_N^2(1+\theta_N)}, \mu_3' = \frac{3(5+2\theta_N)}{\theta_N^3(1+\theta_N)}, \text{ and } ,\mu_4' = \frac{24(3+\theta_N)}{\theta_N^4(1+\theta_N)}.$$

Using the r-th raw moment, we can determine the p-th central moments of X_N as follows

$$\mu_p = E(X_N-\mu_1')^p = \sum_{\ell=0}^{p}(-1)^\ell \binom{p}{\ell}(\mu_1')^{p-\ell}\mu_{p-\ell}'.$$

We used μ_r', $r=1,\ldots,4$, to obtain the variance, coefficient of variation (CV), skewness (γ_1), and kurtosis (γ_2). They are given as follows:

$$\begin{aligned}
Var(X_N) &= \frac{(4\theta_N^2+12\theta_N+7)}{4\theta_N^2(\theta_N+1)^2},\\
CV &= \frac{\sqrt{Var(X_N)}}{\mu_1'} = \frac{\sqrt{4\theta_N^2+12\theta_N+7}}{3+2\theta_N},\\
\gamma_1 &= \frac{\mu_3'-3\mu_2'\mu_1'+2(\mu_1')^3}{(\mu_2'-(\mu_1')^2)^{3/2}} = \frac{2(8\theta_N^3+36\theta_N^2+42\theta_N+15)}{(4\theta_N^2+12\theta_N+7)^{3/2}},
\end{aligned}$$

and

$$\gamma_2 = \frac{\mu_4'-4\mu_3'\mu_1'+6\mu_2'(\mu_1')^2-3(\mu_1')^4}{(\mu_2'-(\mu_1')^2)^2} = \frac{144\theta_N^4+864\theta_N^3+1608\theta_N^2+1224\theta_N+333}{(4\theta_N^2+12\theta_N+7)^2}.$$

For $\theta_N>0$, the expressions for $\mathrm{Var}(X_N)$, CV, γ_1, and γ_2 are differentiable with respect to θ_N. We have

$$\begin{aligned}
\frac{d\mu_1'}{d\theta_N} &= -\frac{2\theta_N^2+6\theta_N+3}{2\theta_N^2(1+\theta_N)^2},\\
\frac{dVar(X_N)}{d\theta_N} &= -\frac{4\theta_N^3+18\theta_N^2+20\theta_N+7}{2\theta_N^3(\theta_N+1)^3},\\
\frac{dCV}{d\theta_N} &= -\frac{10\theta_N+9}{(3\theta_N+2)^2\sqrt{4\theta_N^2+12\theta_N+7}},\\
\frac{d\gamma_1}{d\theta_N} &= \frac{96\theta_N^2+144\theta_N+48)}{(4\theta_N^2+12\theta_N+7)^{5/2}},
\end{aligned}$$

and

$$\frac{d\gamma_2}{d\theta_N} = \frac{192(8\theta_N^3+18\theta_N^2+13\theta_N+3)}{(4\theta_N^2+12\theta_N+7)^3}.$$

Since θ_N is positive, $\frac{d\mu_1'}{d\theta_N}$, $\frac{dCV}{d\theta_N}$ and $\frac{dVar(X_N)}{d\theta_N}$ are strictly negative while $\frac{d\gamma_1}{d\theta_N}$ and $\frac{d\gamma_2}{d\theta_N}$ are strictly positive. These imply that $\mathrm{Var}(X_N)$ and CV are decreasing in θ_N while γ_1, and γ_2 are increasing. Table 1 contains

Table 1: Mean, variance, coefficient of variation, skewness and kurtosis of the NZLD.

θ_N	μ_1'	$Var(X_N)$	CV	γ_1	γ_2
[0.10,0.25]	[5.6,14.545]	[26.24,170.248]	[0.897,0.915]	[1.655,1.699]	[6.957,7.173]
[0.25,0.50]	[2.667,5.600]	[6.222,26.24]	[0.915,0.935]	[1.699,1.756]	[7.173,7.469]
[0.80,1.00]	[1.250,1.597]	[1.438,2.310]	[0.952,0.959]	[1.806,1.831]	[7.743,7.888]
[1.25,1.75]	[0.675,0.978]	[0.434,0.893]	[0.966,0.976]	[1.856,1.892]	[8.038,8.259]
[2.00,2.50]	[0.457,0.583]	[0.202,0.326]	[0.979,0.984]	[1.906,1.925]	[8.343,8.472]
[2.50,3.50]	[0.375,0.457]	[0.137,0.202]	[0.984,0.988]	[1.925,1.94]	[8.472,8.567]

the values of variance, CV, skewness (γ_1), and kurtosis (γ_2) for some values selected of θ_N under the neutrosophic framework. Table 1 shows that the mean, variance, and coefficient of variation decrease as θ_N increases, whereas the skewness and kurtosis increase. This behavior is also evident from their formulas as θ_N increases.

4 MLE of neutrosophic parameter

Let $x_1, x_2, \ldots, x_n$ be observed values of $X_1, X_2, \ldots, X_n$, where $X_1, \ldots, X_n$ are independent random variables following the ZLindley distribution with unknown parameter θ. Then, the log-likelihood function is

$$\ln L(\theta; x_1, \ldots, x_n) = n \ln \theta - n \ln(2) - n \ln(\theta + 1) + \sum_{i=1}^{n} \ln(1 + 2\theta + \theta x_i) - \theta \sum_{i=1}^{n} x_i.$$

The MLE of θ is obtained by solving the non-linear equation $\frac{\partial}{\partial \theta} \ln L(\theta; x_1, \ldots, x_n) = 0$, where

$$\frac{\partial}{\partial \theta} \ln L(\theta; x_1, \ldots, x_n) = \frac{n}{\theta(\theta+1)} + \sum_{i=1}^{n} \frac{2 + x_i}{1 + 2\theta + \theta x_i} - \sum_{i=1}^{n} x_i. \tag{4}$$

We extend the maximum likelihood framework to account for the uncertainty of the data by defining each observation as an interval $x_{Ni} = [x_{Li}, x_{Ui}]$ for $i = 1, \ldots, n$. Consequently, the neutrosophic log-likelihood function is

$$\ln L(\theta_N; x_{N1}, \ldots, x_{Nn}) = n \ln(\theta_N) - n \ln(2) - n \ln(\theta_N + 1) + \sum_{i=1}^{n} \ln\big(1 + 2\theta_N + \theta_N x_{Ni}\big) - \theta_N \sum_{i=1}^{n} x_{Ni}.$$

In neutrosophic statistics, the likelihood function is optimized over the interval space of the data to account for indeterminacy. Consequently, we use a two-step interval maximization to obtain the neutrosophic MLE (NMLE):

- The first step is to define the indeterminate space $S_N = \prod_{i=1}^{n} x_{Ni}$ which is the space of all possible observations, where each observation interval x_{Ni} adds to the total uncertainty of the parameter.
- The second step is to determine the MLE of parameters for each $\mathbf{x} = (x_{N1}, \ldots, x_{Nn}) \in S_N$ using equation (4). The minimum and maximum estimates for each of the possible values of $\mathbf{x}$ are then calculated to get the NMLE.

For a given interval observation $\mathbf{x} \in S_N$. Let $\Theta(\mathbf{x}) = \arg\max_{\theta_N} \ln L(\theta, x_{N1}, \ldots, x_{Nn})$ represents the set of parameter values that maximize the likelihood function. Therefore, the following formula can be used to calculate the NMLE:

$$\hat{\theta}_N = \left(\min_{x \in S_N} \{\theta : \theta \in \Theta(x)\},\ \max_{x \in S_N} \{\theta : \theta \in \Theta(x)\} \right). \tag{5}$$

A large number of data vectors are generated within the admissible region S_N to carry out the nonlinear optimization in (4). Each vector is created by independently sampling values from the observed intervals. For each generated vector, the corresponding MLE is recorded after maximizing the likelihood function with respect to the parameter. Finally, the extreme values of these estimates across all evaluated instances are extracted to obtain $\hat{\theta}_N$.

5 Simulation study

The performance of NMLE in the NZLD is assessed through a simulation study. The NZLD provides interval-valued estimates that account for both inherent uncertainty and randomness, in contrast to the point estimates provided by classical statistics. A neutrosophic variable is defined as $X_N = X + I$, where X is the determined part, and I is the uncertain part. Within this framework, I can be represented as a uniform distribution $I \sim U(-\varepsilon, \varepsilon)$ to represent variable uncertainty, or it can be a constant value that captures fixed uncertainty. This method improves the accuracy of real-world estimations by allowing the model to incorporate both static and dynamic uncertainties. In our simulation study, we generate $N = 1000$ random samples of sizes $n = 10, 20, 50, 200$, and 300 from NZLD with $\theta = 1.25$ and $\theta = 2$. Additive uniform noise $I \sim U(-\varepsilon, \varepsilon)$ was used to introduce uncertainty for each observation x_N, with $\varepsilon = 0.01, 0.05, 0.1$ and 0.2. This approach reflects the model's adaptability to uncertain data and captures variability. Using (5), we compute NMLE to evaluate the performance. To this end, we calculate the neutrosophic mean squared error (NMSE), neutrosophic average estimates (NAE) and neutrosophic average biases (NAB):

$$\text{NAE} = \frac{1}{1000}\sum_{i=1}^{1000} \theta_{N_i}, \;\; \text{NAB} = \frac{1}{1000}\sum_{i=1}^{1000} (\theta_{N_i} - \theta_N), \;\; \text{and} \;\; \text{MSE} = \frac{1}{1000}\sum_{i=1}^{1000} (\theta_{N_i} - \theta_N)^2 .$$

Tables 2 and 3 confirm that the estimator of the parameter of the NZLD works consistently with more

Table 2: NAE, NAB and NMSE for θ =1.25.

ε	n	NAE	NAB	NMSE
0.01	10	[1.3424,1.3498]	[0.0924,0.0998]	[0.4376,0.4454]
	50	[1.2575,1.2634]	[0.0075,0.0134]	[0.1537,0.1557]
	100	[1.2536,1.2594]	[0.0036,0.0094]	[0.1053,0.1066]
	300	[1.2522,1.258]	[0.0022,0.0080]	[0.0608,0.0618]
0.05	10	[1.328,1.3653]	[0.0780,0.1153]	[0.4225,0.4620]
	50	[1.2459,1.2754]	[-0.0041,0.0254]	[0.1505,0.1604]
	100	[1.2422,1.2712]	[-0.0078,0.0212]	[0.1035,0.1104]
	300	[1.2409,1.2697]	[-0.0091,0.0197]	[0.0603,0.0656]
0.1	10	[1.3107,1.3853]	[0.0607,0.1353]	[0.4055,0.4850]
	50	[1.2317,1.2908]	[-0.0183,0.0408]	[0.1480,0.1678]
	100	[1.2282,1.2863]	[-0.0218,0.0363]	[0.1032,0.1170]
	300	[1.227,1.2847]	[-0.023,0.0347]	[0.0626,0.0730]
0.2	10	[1.2779,1.4283]	[0.0279,0.1783]	[0.3771,0.5411]
	50	[1.2044,1.3229]	[-0.0456,0.0729]	[0.1474,0.1869]
	100	[1.2011,1.3175]	[-0.0489,0.0675]	[0.1082,0.1354]
	300	[1.2001,1.3158]	[-0.0499,0.0658]	[0.0748,0.0945]

data and effectively reduces estimation error by demonstrating that NMSE decreases with larger sample sizes. Moreover, the parameter intervals widen as ε increases, indicating the flexibility of the model in the face of increased uncertainty. This procedure provides more accurate estimates in the face of data indeterminacy and is consistent with the behavior predicted by the interval neutrosophic framework.

Table 3: NAE, NAB and NMSE for θ =2.

ε	n	NAE	NAB	NMSE
0.01	10	[2.1509,2.1714]	[0.1509,0.1714]	[0.7165,0.7389]
	50	[2.0096,2.0257]	[0.0096,0.0257]	[0.2499,0.2554]
	100	[2.0031,2.0189]	[0.0031,0.0189]	[0.1711,0.1749]
	300	[2.0006,2.0163]	[0.0006,0.0163]	[0.0987,0.1017]
0.05	10	[2.1121,2.2147]	[0.1121,0.2147]	[0.6758,0.7881]
	50	[1.9783,2.0589]	[-0.0217,0.0589]	[0.2423,0.2700]
	100	[1.9722,2.0513]	[-0.0278,0.0513]	[0.1680,0.1872]
	300	[1.9700,2.0486]	[-0.0300,0.0486]	[0.1002,0.1148]
0.1	10	[2.0663,2.2727]	[0.0663,0.2727]	[0.6332,0.8629]
	50	[1.9407,2.1021]	[-0.0593,0.1021]	[0.2393,0.2945]
	100	[1.9351,2.0935]	[-0.0649,0.0935]	[0.1722,0.2103]
	300	[1.9331,2.0905]	[-0.0669,0.0905]	[0.1137,0.1416]
0.2	10	[1.9825,2.4051]	[-0.0175,0.4051]	[0.5709,1.0877]
	50	[1.8700,2.1949]	[-0.1300,0.1949]	[0.2512,0.3626]
	100	[1.8651,2.1837]	[-0.1349,0.1837]	[0.2006,0.2771]
	300	[1.8635,2.1801]	[-0.1365,0.1801]	[0.1611,0.2164]

6 Applications

In this section, two data sets are used to illustrate the applicability of the proposed distribution.

6.1 Pseudo neutrosophic data

In order to prove the potentiality of the NZLD, we give an application using a well-known real data set. The dataset is given by Ghitany el al. (2008), and represents the waiting times (in minutes) before service of 100 bank customers. The dataset is:
0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7 ,6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1 and 38.5.

We use the parameter ε to introduce different levels of indeterminacy in order to extend the analysis into the neutrosophic framework. The exact dataset is used to build neutrosophic datasets, in which a certain amount of uncertainty is introduced by each ε value. To represent varying degrees of indeterminacy, we use $\varepsilon = 0, 0.001, 0.01, 0.1$ and 1 and take into consideration $X_N = X + \varepsilon$. The data is accurate and represents a classical situation when $\varepsilon = 0$. The dataset shows increasing degrees of uncertainty as ε increases, which is consistent with the neutrosophic notion. We estimate the parameter θ_N of the NZLD model across the various ε-values using the transformed datasets. Table 3 displays these estimates as well as the corresponding Bayesian Information Criterion (BIC) and Akaike's Information Criterion (AIC).

To extend the analysis into the neutrosophic framework, we introduce varying levels of indeterminacy using the parameter ε. The precise dataset serves as the basis for creating neutrosophic datasets, where each ε value introduces a specific degree of uncertainty. We take $X_N = X + \varepsilon$ and set $\varepsilon = 0, 0.001, 0.01, 0.1$, and 0.75 to show different levels of indeterminacy. When $\varepsilon = 0$, the dataset remains precise, reflecting a classical scenario. As ε increases, the dataset captures increasing levels of uncertainty, aligning with the neutrosophic concept.
Using the transformed datasets, we estimate the parameter θ of the NZLD model for different ε-values.

This estimate, the Akaike's Information Criterion (AIC), and the Bayesian Information Criterion (BIC) are presented in table 4. The AIC and BIC interval values reveal how well the neutrosophic model fits the data at various levels of uncertainty. The neutrosophic model approaches the behavior of classical statistical models, which rely on exact data, as the degree of uncertainty decreases (i.e., smaller ε-values). This specifically corresponds to the ZLD when $\varepsilon = 0$. However, the model captures more uncertainty with larger ε-values, indicating the adaptability and resilience of the neutrosophic approach. Since θ_N is unknown, we evaluate the model's fit using the modified Kolmogorov-Smirnov (KS*) test.

Table 4: NAE, NAB and NMSE for θ =1.25.

ε	θ_N	Log-likelihood	AIC	BIC	KS*
0	0.1481	-325.2583	652.5167	655.1219	0.5076
0.001	[0.14805,0.14808]	[-325.267,-325.250]	[652.499,652.534]	[655.104,655.140]	[0.5075,0.5078]
0.01	[0.1479,0.1482]	[-325.346,-325.171]	[652.342,652.692]	[654.947,655.297]	[0.506,0.509]
0.1	[0.1468,0.1494]	[-326.131,-324.379]	[650.759,654.263]	[653.364,656.866]	[0.491,0.523]
0.75	[0.1387,0.1587]	[-331.641,-318.495]	[638.989,665.282]	[641.595,667.887]	[0.599,0.610]

6.2 Neutrosophic failure time dataset

This dataset, provided by Linhart and Zucchini (1986, p. 69), represents the failure times of the air conditioning system of an airplane. The corresponding neutrosophic data are:
23, 261, 87, 7, [121,125], 14, 62, 47, 225, [71,74], 246, 21, 42, 20, 5, 12, 120, [11,13], 3, 14, 71, 11, 14, 11, 16, 90, 1, 16, [52,55] and 95.

We compare the performance of the NZLD with the neutrosophic Lindley distribution (NLD), proposed by Bashir el al. (2025). From Table 5, the NZLD model provides the lowest AIC, BIC, CAIC, HQIC, and KS* values, indicating superior performance compared to the NLD.

Table 5: Comparison between the NZLD and NLD.

Models	NZLindley	NLindley
NMLE of θ_N	[0.0238,0.0240]	[0.0328,0.0330]
Log-L	[-154.508,-154.338]	[-161.8429,-161.5933]
AIC	[310.676,311.017]	[325.1866,325.6859]
BIC	[312.077,312.418]	[326.5878,327.0871]
CAIC	[313.077,313.418]	[327.5878,328.0871]
HQIC	[311.124,311.465]	[325.6348,326.1341]
KS*	[0.0199,0.0234]	[0.0240,0.0269]
p-value	[0.9670,0.9820]	[0.9527,0.9679]

7 Conclusion

Within a neutrosophic framework, we introduced an extension of the ZLindley distribution, termed the neutrosophic ZLindley distribution (NZLD). Several properties of the proposed model were derived, and the model parameter was estimated using NMLE. A simulation study demonstrated that this estimator performs best under both bias and MSE criteria. Additionally, a well-known real dataset was analyzed, and the results indicate that the proposed model provides a superior fit compared to the NLD.

References

Alduais, F. S., and Khan, Z. (2025). Development of neutrosophic gamma distribution for modeling neonatal mortality data. *Neutrosophic Sets and Systems*, **79**, 48.

Alhabib, R., Ranna, M. M., Farah, H. and Salama, A. A. (2018). Some neutrosophic probability distributions. *Neutrosophic Sets and Systems*, **22**, 30–38.

Alhasan, K. F. H. and Smarandache, F. (2019). Neutrosophic Weibull distribution and neutrosophic family Weibull distribution. *Neutrosophic Sets and Systems*, **28**, 191–199.

Bashir, S., Masood, B., Shehzadi, I. and Aslam, M. (2024). Neutrosophic log-Logistic distribution: a novel approach for modeling uncertain survival and reliability data. *REVSTAT-Statistical Journal.* Accepted-November 2024. https://revstat.ine.pt/index.php/REVSTAT/article/view/789.

Bashir, S., Masood, B., Shehzadi, I., Al-Husseini, Z. and Aslam, M. (2025). Neutrosophic Lindley distribution: simulation, application, and comparative study. *Contemporary Mathematics*, **2025**, 551–564.

Ghitany, M. E, Atieh, B. and Nadarajah, S. (2008). Lindley distribution and its application. *Mathematics and Computers in Simulation*, **78**, 493–506.

Khan Sherwani, R.A., Naeem, M., Aslam, M., and Raza, M. A. (2021). Neutrosophic beta distribution with properties and applications. *Neutrosophic Sets and Systems*, **41**, 12–19.

Linhart, H. and Zucchini, W. (1986). *Model Selection.* John Wiley, New York.

Razmkhah, M, Arashi, A., Bekker, A. and Marques, F. J. (2026). Neutrosophic Birnbaum-Saunders distribution with applications. *Applied Mathematical Modelling* , **149**, 116287.

Saaidia, N., Belhamra, T. and Zeghdoudi, H. (2024). On ZLindley distribution: statistical properties and applications. *Studies in Engineering and Exact Sciences*, **5**, 3078–3097.

Smarandache, F. (2014). *Introduction to Neutrosophic Statistics.* Sitech and Education Publishing, Columbus, Ohio.