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On the use of a family of exponential-type estimators under equal probability sampling scheme in presence of measurement error

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Abstract. This study develops a class of estimators for estimating the population mean in survey data under measurement error. The analysis is conducted under simple random sampling without replacement (SRSWOR), with attention given to two distinct measurement-error models: the Constant Correlation Model (CCM) and the Inter-sample Correlation Model (ICM). The bias and mean squared error (MSE) of the proposed estimators are derived to assess their statistical performance. The efficiency of the proposed estimators is examined against standard estimators to evaluate relative efficiency under measurement error in the observed data. The empirical findings indicate that the proposed estimator performs comparatively better than the competing estimators, including the ratio estimator. In particular, it provides more efficient and reliable estimates of the population mean under measurement errors. The proposed methodology may also prove useful in reliability and survival studies, especially when recorded lifetimes or failure times of items or individuals are subject to measurement error.

Keywords: Bias; CCM; ICM; Measurement error; MSE.

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1 Introduction

Estimating the average value of a population is a common objective in survey sampling and applied research. Researchers may be interested in quantities such as average income, average production, average lifetime, or average survival time. In many studies, information about a second variable connected to the main variable is also available. This additional variable, known as an auxiliary variable, can help optimise the precision of the estimate. In this paper, the estimation procedure is formulated within the framework of SRSWOR. This means that each component of the population has the same possibility of being selected, and once a member is selected, it is not selected again.

In real-life situations, the recorded data may not be exactly correct. The value written in a questionnaire, measured by an instrument, or entered into a computer may differ from the actual value. These situations lead to two important classifications of non-sampling errors: (i) Measurement Error and (ii) Non-Response Error.

Measurement error occurs when the information recorded in a survey is not the same as the actual value of the unit being studied. This error may arise from the respondent, the interviewer, or the method used to collect the information. For example, in an agricultural survey, a farmer may report the total area of his land, but the reported figure may differ from the area given in the official land records. In these situations, measurement errors can affect the accuracy of commonly used statistical methods and may influence the conclusions drawn from survey data. When these errors are very small, they may have little effect on the results. However, some level of error is common in survey data. Shalabh (2000) observed that if the measurement errors are not negligible, they may produce inaccurate or misleading results and can sometimes lead to unexpected conclusions. Therefore, it is important to take measurement errors into account when drawing statistical conclusions, as they may influence the accuracy of the results. An extensive discussion of measurement errors has been provided by Fuller (1987). Biemer (2009) discussed ME in sample surveys and their effects on survey results.

In survey investigations, it is commonly accepted that the information obtained regarding the target variable (Z) and the auxiliary variable (X) accurately depicts the true values of the observations. In reality, however, this presumption may not always hold because of errors that can arise during data collection, recording, or measurement. Such errors are referred to as measurement errors (ME). The variation between the real and reported value of a variable is termed measurement error. The impact of measurement error in survey sampling was investigated by Murthy (1967) and Cochran (1968). Cheng and Van Ness (1994), Fuller (1987) and Carroll (1995) also addressed how ME can affect the performance of estimators. Biemer and Stokes (1991) noted that measurement errors can sometimes result in misleading conclusions. Many studies in survey sampling have also examined the use of an additional variable to optimise the performance of population parameters. Among these, regression, product, and ratio estimators have been widely studied. Several researchers, such as Gregoire and Salas (2009), Kumar et al. (2011), Sahoo et al. (2006), Shalabh (1997), and Manisha and Singh (2001), have analysed the effects of measurement errors on ratio and regression estimators used to estimate the population mean.

Hansen and Hurwitz (1946) suggested a framework for non-response by making extra efforts to collect information from some of the non-responding units. El-Badry (1956) later extended

this approach. Khare and Srivastava (1993) developed a ratio and product estimation method for the population mean using two-phase sampling and studied their performance by taking a portion of non-respondent participants. Khare and Srivastava (1997) also worked in this area, and their work was later discussed by Tabasum and Khan (2004). A set of population mean estimators was developed by Sharma and Singh (2013) using point bi-serial and phi correlation coefficients. More recently, Singh et al. (2024) used a design-based approach with auxiliary information to improve population mean estimation when non-response occurs. The primary focus of these studies is non-response, but measurement error may also have an impact on the precision of survey findings.

In this paper, we suggest a group of estimators for calculating the population mean under ME. This research is carried out using SRSWOR and considers two measurement-error models: the constant correlation model (CCM) and the inter-sample correlation model (ICM). These models allow us to study different ways in which measurement errors may be related. We derive expressions for the bias and MSE of the suggested estimators and then compare them with commonly used estimators, including the ratio estimator. The objective of this research is to determine whether the suggested estimators can give more reliable estimates than standard methods under measurement error. This study primarily focuses on estimating the average value of a finite population. It does not directly cover other topics in survival analysis, such as censored data, truncated data, or hazard rates. These topics can be studied in future research. However, the proposed estimators may also be useful in reliability and survival studies. Reliability studies often look at how long a machine or device works before it stops working, while survival studies look at how long a person or subject remains alive or free from a particular event. In both cases, the recorded time may not always be accurate because of measurement or recording errors. For example, the exact time when a machine stops working may not be recorded correctly, or the time of an event may be reported incorrectly. In such situations, the proposed estimators may help provide more reliable estimates of the average lifetime when ME are present.

2 Origin of the problem

Let us assume a finite population of N distinct units. Let Z represent the target variable and X represent the auxiliary information, both of which are measured. Let (z_i, x_i), $i = 1,2,\ldots,N$, represent the pair of values of Z and X on the i^{th} unit of the population.

A sample of size n is selected from the population according to a specified sampling scheme. The sample of size n is denoted by s. The pairs $(\bar{Z}, \bar{X})$ and $(\bar{z}, \bar{x})$ stand for the population means and sample means, respectively, of the study and auxiliary variables. Therefore, we presume that measurement error is present exclusively in the study variable, Z. Under this assumption,

$$z_{jk} = z_j + \mu_j + e_{jk}, \qquad j = 1,2,\ldots,N,$$

where z_{jk} is the value of the j^{th} unit on the k^{th} repetition, z_j represents the true value of the j^{th} unit, μ_j is the instrumental error occurring on the j^{th} unit, and e_{jk} is the measurement error occurring on the j^{th} unit in the k^{th} repetition.

3 ME models under consideration

Based on the ideas of Chandhok and Han (1990) and Kumar et al. (2019), this work considers two measurement-error models: the Constant Correlation Model (CCM) and the Inter-sample Correlation Model (ICM). Both models operate on the assumption that measurement errors from repeated observations of the same unit are not linked. The structure of the error covariance for distinct units, meanwhile, varies among the models. In the CCM, this covariance is represented by the correlation coefficient ρ and the standard deviations of the significant measurement errors. In the ICM, covariance is determined using the measurement-error variance defined by the model and the average inter-sample correlation coefficient, ρ_w. The ICM framework likewise articulates the association between ρ_w and ρ. The bias and mean squared error (MSE) of the proposed estimators are analysed in relation to these two measurement-error contexts using both models. We consider here two measurement-error models.

3.1 Constant correlation model (CCM)

Let us assume

$$z_{jk} = z_j + \mu_j + e_{jk} = z'_j + e_{jk}; \quad j = 1,2,\ldots,N,$$

where

$$E[e_{jk} \mid j] = 0, \quad E[e_{jk}^2 \mid j] = V[e_{jk} \mid j] = \sigma_j^2, \quad \mathrm{Cov}[e_{jk}, e_{jk'} \mid j] = 0 \quad (k \neq k'),$$

and

$$\mathrm{Cov}[e_{jk}, e_{j'k} \mid j, j'] = \rho\sigma_j\sigma_{j'} \quad (j \neq j').$$

Under this model, the error terms for the same unit across various trials are presumed to be non-correlated, whereas error terms for different units are correlated with a correlation coefficient ρ.

3.2 Inter-sample correlation model (ICM)

The average inter-sample correlation coefficient ρ_w can be used, as in cluster sampling, to define a more general measurement-error model as

$$z_{jk} = z_j + \mu_j + e_{jk} = z'_j + e_{jk}; \quad j = 1,2,\ldots,N,$$

where

$$E[e_{jk} \mid j] = 0, \quad E[e_{jk}^2 \mid j] = V[e_{jk} \mid j] = \sigma_j^2, \quad \mathrm{Cov}[e_{jk}, e_{jk'} \mid j] = 0 \quad (k \neq k'),$$

and

$$\mathrm{Cov}[e_{jk}, e_{j'k} \mid j, j'] = \rho_w\sigma_j^2 \quad (j \neq j').$$

The relation between ρ_w and ρ is given by

$$\rho_w = \frac{\sum_{j=1}^{N}\sum_{\substack{j'=1 \\ j'\neq j}}^{N}\sigma_j\sigma_{j'}}{(N-1)\sum_{j=1}^{N}\sigma_j^2\rho}.$$

4 Notations and assumptions

The notations used throughout this study are defined as follows:

$$\theta = \left(\frac{1}{n} - \frac{1}{N}\right), \quad \bar{z}'_n = \frac{1}{n}\sum_{j=1}^{n} z'_j, \quad \bar{z}'_N = \frac{1}{N}\sum_{j=1}^{N} z'_j, \quad \bar{\mu}_N = \frac{1}{N}\sum_{j=1}^{N} \mu_j, \quad C^2_{z'} = \frac{\sum_{j=1}^{N}(z'_j - \bar{z}'_N)^2}{(N-1)(\bar{z}'_N)^2},$$

$$C^2_x = \frac{\sum_{j=1}^{N}(x_j - \bar{X})^2}{(N-1)\bar{X}^2}, \qquad C^2_\sigma = \frac{\sum_{j=1}^{N}(\sigma_j - \bar{\sigma}_N)^2}{(N-1)\bar{\sigma}^2_N}, \qquad C_{xz'} = \frac{\sum_{j=1}^{N}(z'_j - \bar{z}'_N)(x_j - \bar{X})}{(N-1)\bar{z}'_N\,\bar{X}},$$

$$\bar{\sigma}_N = \frac{1}{N}\sum_{j=1}^{N}\sigma_j, \qquad \bar{\sigma}_n = \frac{1}{n}\sum_{j=1}^{n}\sigma_j, \qquad \tau_j = \sigma^2_j, \qquad \bar{\tau}_N = \frac{1}{N}\sum_{j=1}^{N}\sigma^2_j, \qquad \bar{\tau}_n = \frac{1}{n}\sum_{j=1}^{n}\sigma^2_j,$$

$$C_{X\sigma} = \frac{\sum_{j=1}^{N}(x_j - \bar{X})(\sigma_j - \bar{\sigma}_N)}{(N-1)\bar{X}\bar{\sigma}_N} \quad \text{and} \quad C_{X\tau} = \frac{\sum_{j=1}^{N}(x_j - \bar{X})(\tau_j - \bar{\tau}_N)}{(N-1)\bar{X}\bar{\tau}_N}.$$

Remark 1. *The expressions for the bias and mean squared error (MSE) are derived in the theoretical derivations using first-order approximations. The required terms for these expressions are retained and higher-order terms are neglected. The same procedure is used uniformly in all the derivations.*

5 The proposed family of strategies

We denote $[P,T]$ as a sampling strategy, where P stands for the SRSWOR sampling method and T represents the estimator of the population mean $\bar{Z}$. Motivated by Singh (1992) and Bahl and Tuteja (1991), the sampling strategies $[P,T_1(\eta_1,\eta_2)]$ and $[P,T_2(\alpha)]$ are defined, respectively, as

$$T_1(\eta_1,\eta_2) = \eta_1\bar{z} + \eta_2\bar{z}\varphi(\bar{X},\bar{x}),$$

where

$$\varphi(\bar{X},\bar{x}) = \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right),$$

and

$$T_2(\alpha) = \alpha\bar{z} + (1-\alpha)\bar{z}\varphi(\bar{X},\bar{x}),$$

where

$$\varphi(\bar{X},\bar{x}) = \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right).$$

Remark 2. *In the proposed estimator $T_1(\eta_1,\eta_2)$, both η_1 and η_2 are unknown parameters. Since both parameters need to be estimated together, the MSE expression becomes very complicated. Because of this, it is difficult to find their optimal values using standard mathematical methods. Therefore, we used a hit-and-trial method. In this method, different values of η_1 and η_2 were tried, and the combination that gave the smallest MSE was selected.*

To make the estimation process easier, the estimator $T_2(\alpha)$ was introduced. In this model, we set $\eta_1=\alpha$ and $\eta_2=1-\alpha$, where only α is unknown. This reduced the number of undefined parameters from two to one, making the optimisation process much simpler. The optimum value of α could then be obtained more easily, and the corresponding minimum MSE was calculated. The results showed that $T_2(\alpha)$ gives better performance than $T_1(\eta_1,\eta_2)$ because it produces a smaller MSE and is easier to optimise.

6 Bias and MSE expressions under the SRSWOR scheme

Theorem 1. *The bias and MSE of the estimator $T_1(\eta_1,\eta_2)$, up to first-order approximations under the CCM model, are given respectively as*

$$B_C[T_1(\eta_1,\eta_2)]=\bar{\mu}_N+(\bar{Z}+\bar{\mu}_N)\eta_2\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)\theta,$$

and

$$\begin{aligned}M_C[T_1(\eta_1,\eta_2)]&=(\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2+\frac{\eta_2^2C_x^2}{4}-\eta_2C_{xz'}\right)\theta\\&\quad+\rho(\bar{\sigma}_N)^2\left(1+\frac{\eta_2^2\theta C_x^2}{4}+\frac{3}{4}\eta_2\theta C_x^2+\theta C_\sigma^2-\eta_2\theta C_{x\sigma}\right)+\bar{\mu}_N^2\\&\quad+\bar{\tau}_N\left(\frac{1-\rho}{n}\right)\left(1+\frac{\eta_2^2\theta C_x^2}{4}+\frac{3}{4}\eta_2\theta C_x^2-\eta_2\theta C_{x\tau}\right)+2\eta_2(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)\theta.\end{aligned}$$

Proof. Let us first consider the CCM. Then we have

$$\begin{aligned}B_C[T_1(\eta_1,\eta_2)]&=E_1E_2\left[T_1(\eta_1,\eta_2)-\bar{Z}\right]\\&=E_1E_2\left[(\eta_1\bar{z}+\eta_2\bar{z}\varphi(\bar{X},\bar{x}))-\bar{Z}\right].\end{aligned}$$

Since $E_2(\bar{z})=\bar{z}'_n$, we have

$$B_C[T_1(\eta_1,\eta_2)]=E_1\left[\left(\eta_1\bar{z}'_n+\eta_2\bar{z}'_n\varphi(\bar{X},\bar{x})\right)-\bar{Z}\right].$$

By taking $\bar{z}'_n=\bar{z}'_N(1+d_1)$ and $\bar{x}=\bar{X}(1+d_2)$, we get

$$B_C[T_1(\eta_1,\eta_2)]=E_1\left[\eta_1\bar{z}'_N(1+d_1)+\eta_2\bar{z}'_N(1+d_1)\left(1-\frac{d_2}{2}+\frac{3d_2^2}{8}\right)-\bar{Z}\right].$$

Now writing $\bar{z}'_N=\bar{Z}+\bar{\mu}_N$, we have

$$B_C[T_1(\eta_1,\eta_2)]=\bar{\mu}_N+(\bar{Z}+\bar{\mu}_N)\eta_2\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)\theta.$$

The variance of $T_1(\eta_1,\eta_2)$ under the CCM will be

$$V_C[T_1(\eta_1,\eta_2)] = V_1E_2[T_1(\eta_1,\eta_2)] + E_1V_2[T_1(\eta_1,\eta_2)],$$

where V_2 denotes the variance over repeated measurements given the sample and V_1 the variance over all possible samples. We have

$$\begin{aligned} V_1E_2[T_1(\eta_1,\eta_2)] &= V_1E_2\left[\eta_1\bar{z} + \eta_2\bar{z}\varphi(\bar{X},\bar{x})\right] \\ &= V_1\left[\eta_1\bar{z}'_n + \eta_2\bar{z}'_n\varphi(\bar{X},\bar{x})\right] \\ &= E_1\left[\eta_1\bar{z}'_n + \eta_2\bar{z}'_n\varphi(\bar{X},\bar{x}) - \bar{Z}'_N\right]^2. \end{aligned}$$

Using the large-sample approximations, we have

$$V_1E_2[T_1(\eta_1,\eta_2)] = (\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2 + \frac{\eta_2^2C_x^2}{4} - \eta_2C_{xz'}\right)\theta.$$

Further,

$$\begin{aligned} E_1V_2[T_1(\eta_1,\eta_2)] &= E_1E_2\left[T_1(\eta_1,\eta_2) - E_2(T_1(\eta_1,\eta_2))\right]^2 \\ &= E_1E_2\left[\eta_1(\bar{z}-\bar{z}'_n) + \eta_2\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)(\bar{z}-\bar{z}'_n)\right]^2. \end{aligned} \tag{1}$$

Now realizing that

$$E_2(z_{jk}) = z'_j, \quad E_2(z_{jk}^2) = z_j'^2 + \sigma_j^2, \quad \text{and } E_2(z_{jk},z_{j'k}) = \rho\sigma_j\sigma_{j'} + z'_jz'_{j'},$$

we have

$$E_2(\bar{z}-\bar{z}'_n)^2 = \frac{(1-\rho)}{n}\bar{\tau}_n + \rho(\bar{\sigma}_n)^2. \tag{2}$$

Substituting (2) into (1), we have

$$E_1V_2[T_1(\eta_1,\eta_2)] = E_1\left[\left(\eta_1 + \eta_2\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right)^2\left(\frac{(1-\rho)}{n}\bar{\tau}_n + \rho(\bar{\sigma}_n)^2\right)\right]. \tag{3}$$

Finally, evaluating E_1 in (3) using Taylor's expansion, we get

$$\begin{aligned} E_1V_2[T_1(\eta_1,\eta_2)] &= \rho(\bar{\sigma}_N)^2\left(1 + \frac{\eta_2^2\theta C_x^2}{4} + \frac{3}{4}\eta_2\theta C_x^2 + \theta C_\sigma^2 - \eta_2\theta C_{x\sigma}\right) \\ &\quad + \bar{\tau}_N\left(\frac{1-\rho}{n}\right)\left(1 + \frac{\eta_2^2\theta C_x^2}{4} + \frac{3}{4}\eta_2\theta C_x^2 - \eta_2\theta C_{x\tau}\right). \end{aligned}$$

Therefore, the MSE of $T_1(\eta_1,\eta_2)$ under the CCM is

$$\begin{aligned}
M_C[T_1(\eta_1,\eta_2)] &= V_C[T_1(\eta_1,\eta_2)] + [B_C(T_1(\eta_1,\eta_2))]^2 \\
&= (\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2 + \frac{\eta_2^2 C_x^2}{4} - \eta_2 C_{xz'}\right)\theta \\
&+ \rho(\bar{\sigma}_N)^2\left(1 + \frac{\eta_2^2\theta C_x^2}{4} + \frac{3}{4}\eta_2\theta C_x^2 + \theta C_\sigma^2 - \eta_2\theta C_{x\sigma}\right) \\
&+ \bar{\tau}_N\left(\frac{1-\rho}{n}\right)\left(1 + \frac{\eta_2^2\theta C_x^2}{4} + \frac{3}{4}\eta_2\theta C_x^2 - \eta_2\theta C_{x\tau}\right) \\
&+ \bar{\mu}_N^2 + 2\eta_2(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right)\theta.
\end{aligned}$$

□

Following the same approach, the bias and MSE of the proposed estimator can be expressed under the ICM model.

Theorem 2. *The bias and MSE of the estimator $T_1(\eta_1,\eta_2)$, up to first-order approximations under the ICM model, are given by, respectively*

$$B_I[T_1(\eta_1,\eta_2)] = B_C[T_1(\eta_1,\eta_2)],$$

and

$$\begin{aligned}
M_I[T_1(\eta_1,\eta_2)] &= (\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2 + \frac{\eta_2^2 C_x^2}{4} - \eta_2 C_{xz'}\right)\theta \\
&+ \left(\frac{n-1}{n}\right)\rho_w\bar{\tau}_N + \frac{\bar{\tau}_N}{n}\left(1 + \frac{\eta_2^2\theta C_x^2}{4} + \frac{3}{4}\eta_2\theta C_x^2 - \eta_2\theta C_{x\tau}\right) \\
&+ \bar{\mu}_N^2 + 2\eta_2(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right)\theta.
\end{aligned}$$

The bias and MSE of the proposed estimator $T_2(\alpha)$ under the CCM and ICM models can be obtained in the same manner as those derived in Theorem 1 and Theorem 2.

Theorem 3. *The bias and MSE of the estimator $T_2(\alpha)$, up to first-order approximations under the CCM and ICM models, are given as*

$$B_C[T_2(\alpha)] = B_I[T_2(\alpha)] = \bar{\mu}_N + (1-\alpha)(\bar{Z}+\bar{\mu}_N)\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right)\theta;$$

$$M_C[T_2(\alpha)] = (\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2+\frac{(1-\alpha)^2C_x^2}{4}-(1-\alpha)C_{xz'}\right)\theta$$
$$+\rho(\bar{\sigma}_N)^2\left(1+\frac{(1-\alpha)^2\theta C_x^2}{4}+2(1-\alpha)\left(\frac{3}{8}\theta C_x^2-\theta C_{x\sigma}\right)+\theta C_\sigma^2\right)$$
$$+\bar{\tau}_N\left(\frac{1-\rho}{n}\right)\left(1+\frac{(1-\alpha)^2\theta C_x^2}{4}+2(1-\alpha)\left(\frac{3}{8}\theta C_x^2-\frac{\theta C_{x\tau}}{2}\right)\right)$$
$$+\bar{\mu}_N^2+2(1-\alpha)(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)\theta,$$

and

$$M_I[T_2(\alpha)] = (\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2+\frac{(1-\alpha)^2C_x^2}{4}-(1-\alpha)C_{xz'}\right)\theta$$
$$+\left(\frac{n-1}{n}\right)\rho_w\bar{\tau}_N+\frac{\bar{\tau}_N}{2}\left(1+\frac{(1-\alpha)^2\theta C_x^2}{4}+2(1-\alpha)\left(\frac{3}{8}\theta C_x^2-\frac{\theta C_{x\tau}}{2}\right)\right)$$
$$+\bar{\mu}_N^2+2(1-\alpha)(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)\theta.$$

The optimum values of α_0 under the CCM and ICM models are obtained, respectively, as

$$[\hat{\alpha}_0]_C=\frac{(\bar{Z}+\bar{\mu}_N)^2\left(\frac{C_x^2}{2}-C_{xz'}\right)+\frac{5}{4}C_x^2\left(\frac{1-\rho}{n}\bar{\tau}_N+\rho(\bar{\sigma}_N)^2\right)+2(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)}{\frac{C_x^2}{2}\left((\bar{Z}+\bar{\mu}_N)^2+\frac{1-\rho}{n}\bar{\tau}_N+\rho(\bar{\sigma}_N)^2\right)},$$

and

$$[\hat{\alpha}_0]_I=\frac{(\bar{Z}+\bar{\mu}_N)^2\left(\frac{C_x^2}{2}-C_{xz'}\right)+\frac{5\bar{\tau}_N}{4n}C_x^2+2(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)}{\frac{C_x^2}{2}\left((\bar{Z}+\bar{\mu}_N)^2+\frac{\bar{\tau}_N}{n}\right)}.$$

7 Particular cases of $T_1(\eta_1,\eta_2)$ and $[T_2(\alpha)]$

The estimators $T_1(\eta_1,\eta_2)$ and $T_2(\alpha)$ are the function of unknown weights and, therefore assigning different values 0 and 1, we have

Weights		$T(\eta_1,\eta_2)$	Weights	$T_2(\alpha)$
η_1	η_2		α	
0	1	$\bar{z}\varphi(\bar{X},\bar{x})$	0	$\bar{z}\varphi(\bar{X},\bar{x})$
1	0	$\bar{z}$	1	$\bar{z}$

The bias and MSE of $T_1(1,0)$ under CCM and ICM, are given by, respectively

$$B_C[T_1(1,0)]=\bar{\mu}_N=B_I[T_1(1,0)],$$

$$M_C[T_1(1,0)] = \theta(\bar{Z}+\bar{\mu}_N)^2 C_{z'}^2 + \rho(\bar{\sigma}_N)^2\left\{1+\theta C_\sigma^2\right\} + \frac{1-\rho}{n}\bar{\tau}_N + (\bar{\mu}_N)^2,$$

and

$$M_I[T_1(1,0)] = \theta(\bar{Z}+\bar{\mu}_N)^2 C_{z'}^2 + \left(\frac{n-1}{n}\right)\rho_w\bar{\tau}_N + \frac{\bar{\tau}_N}{n} + (\bar{\mu}_N)^2.$$

Similarly, the bias and MSE of $T_1(0,1)$ under CCM and ICM, are given by, respectively

$$B_C[T_1(0,1)] = \bar{\mu}_N + \theta(\bar{Z}+\bar{\mu}_N)\left\{\frac{3}{8}C_x^2 - \frac{1}{2}C_{xz'}\right\} = B_I[T_1(0,1)],$$

$$M_C[T_1(0,1)] = \theta(\bar{Z}+\bar{\mu}_N)^2\left\{\frac{C_x^2}{4} + C_{z'}^2 - C_{xz'}\right\} + \rho(\bar{\sigma}_N)^2\left\{1+\theta\left(\frac{C_x^2}{4}+\frac{3}{4}C_x^2+C_\sigma^2-C_{x\sigma}\right)\right\}$$
$$+\bar{\tau}_N\left(\frac{1-\rho}{n}\right)\left\{1+\theta\left(\frac{C_x^2}{4}+\frac{3}{4}C_x^2-C_{x\tau}\right)\right\} + (\bar{\mu}_N)^2 + 2\theta(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left\{\frac{3}{8}C_x^2-\frac{1}{2}C_{xz'}\right\},$$

and

$$M_I[T_1(0,1)] = \theta(\bar{Z}+\bar{\mu}_N)^2\left\{\frac{C_x^2}{4}+C_{z'}^2-C_{xz'}\right\} + \left(\frac{n-1}{n}\right)\rho_w\bar{\tau}_N + \frac{\bar{\tau}_N}{n}\left\{1+\theta\left(\frac{C_x^2}{4}+\frac{3}{4}C_x^2-C_{x\tau}\right)\right\}$$
$$+(\bar{\mu}_N)^2 + 2\theta(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left\{\frac{3}{8}C_x^2-\frac{1}{2}C_{xz'}\right\}.$$

8 Some existing sampling strategies under ME

Singh (1992) suggested the following estimator:

$$T = \eta_1\bar{z} + \eta_2\bar{z}\frac{\bar{X}}{\bar{x}}.$$

The bias and MSE of T under CCM and ICM models are given by, respectively

$$B_C[T] = \bar{\mu}_N + (\bar{Z}+\bar{\mu}_N)\eta_2(C_x^2 - C_{xz'})\theta = B_I[T],$$

$$M_C[T] = (\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2+\eta_2^2C_x^2-2\eta_2C_{xz'}\right)\theta + \rho(\bar{\sigma}_N)^2\left(1+\eta_2^2\theta C_x^2+2\eta_2\theta(C_x^2-2C_{x\sigma})+\theta C_\sigma^2\right)$$
$$+\bar{\tau}_N\left(\frac{1-\rho}{n}\right)\left(1+\eta_2^2\theta C_x^2+2\eta_2\theta(C_x^2-C_{x\tau})\right) + \bar{\mu}_N^2 + 2\eta_2(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N(C_x^2-C_{xz'})\theta,$$

and

$$M_I[T] = (\bar{Z}+\bar{\mu}_N)^2\left(C_{z'}^2+\eta_2^2C_x^2-2\eta_2C_{xz'}\right)\theta + \left(\frac{n-1}{n}\right)\rho_w\bar{\tau}_n + \frac{\bar{\tau}_N}{n}\left(1+\eta_2^2\theta C_x^2+2\eta_2\theta(C_x^2-C_{x\tau})\right)$$
$$+\bar{\mu}_N^2 + 2\eta_2(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N(C_x^2-C_{xz'})\theta.$$

Kumar et al. (2019) proposed the following estimator under measurement error:

$$T_e(\alpha) = \bar{z}\exp\left[\alpha\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right].$$

The bias and MSE $T_e(\alpha)$ under CCM and ICM models are given by, respectively

$$B_C[T_e(\alpha)] = \bar{\mu}_N + \theta\left[\frac{\alpha(\alpha+2)}{8}C_x^2\{\bar{Z}+\bar{\mu}_N\} - \frac{\alpha}{2}C_{xz'}\{\bar{Z}+\bar{\mu}_N\}\right] = B_I[T_e(\alpha)],$$

$$\begin{aligned} M_C[T_e(\alpha)] = {} & (\bar{Z}+\bar{\mu}_N)^2\theta\left\{\frac{\alpha^2}{4}C_x^2 + C_{z'}^2 - \alpha C'_{xz}\right\} + \rho(\bar{\sigma}_N)^2\left[1+\theta\left\{\frac{\alpha(\alpha+1)}{2}C_x^2 + C_\sigma^2 - 2\alpha C_{x\sigma}\right\}\right] \\ & + \frac{1-\rho}{n}\bar{\tau}_N\left[1+\theta\left\{\frac{\alpha(\alpha+1)}{2}C_x^2 - \alpha C_{x\tau}\right\}\right] + (\bar{\mu}_N)^2 \\ & + 2\theta(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left\{\frac{\alpha(\alpha+2)}{8}C_x^2 - \frac{\alpha}{2}C'_{xz}\right\}. \end{aligned}$$

and

$$\begin{aligned} M_I[T_e(\alpha)] = {} & (\bar{Z}+\bar{\mu}_N)^2\theta\left\{\frac{\alpha^2}{4}C_x^2 + C_{z'}^2 - \alpha C'_{xz}\right\} + \frac{\bar{\tau}_N}{n}\left[1+\theta\left\{\frac{\alpha(\alpha+1)}{2}C_x^2 - \alpha C_{x\tau}\right\}\right] \\ & + \left(\frac{n-1}{n}\right)\rho_w\bar{\tau}_N + (\bar{\mu}_N)^2 + 2\theta(\bar{Z}+\bar{\mu}_N)\bar{\mu}_N\left\{\frac{\alpha(\alpha+2)}{8}C_x^2 - \frac{\alpha}{2}C'_{xz}\right\}. \end{aligned}$$

9 Adjustments in the parameters of the model

One may propose particular conditions for the parameters μ_j, σ_j, and ρ in order to track changes in the efficiency of various strategies; it is preferable to alter the values in a systematic way rather than selecting them at random. For simplicity, let us consider the following modifications to the parameters:

(i) The response bias $\mu_j = \beta x_{0j}$ has varying values of $\beta = 0.00, \pm 0.01, \pm 0.05$ and ± 0.10. As a result, differences in response bias are produced by changing these values.

(ii) The variance within trials $\sigma_j^2 = \sigma^2 = \xi\sigma_{X_0}^2$ with $\xi = 0.00, 0.25, 0.50, 0.75$ and 1.00. Over repeats, this gives fluctuations in measurement-error variability, which is constant for all j.

(iii) The correlation coefficient $\rho = 0.00$ and 0.05, respectively.

Remark 3. *It can be seen that the combination of* $\beta = 0.00$, $\xi = 0.00$, *and* $\rho = 0.00$ *denotes the absence of ME. As a result, in terms of* β *and* ξ, *the expressions of* $B_C[T_1(\eta_1,\eta_2)]$, $M_C[T_1(\eta_1,\eta_2)]$ *and* $M_I[T_1(\eta_1,\eta_2)]$ *of the proposed estimator* $T_1(\eta_1,\eta_2)$ *become*

$$B_C[T_1(\eta_1,\eta_2)] = \beta\bar{Z} + \eta_2(1+\beta)\bar{Z}\theta\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right) = B_I[T_1(\eta_1,\eta_2),$$

$$M_C[T_1(\eta_1,\eta_2)] = (1+\beta)^2\bar{Z}^2\theta\left(C_{z'}^2 + \frac{\eta_2^2C_x^2}{4} - \eta_2C_{xz'}\right) + \rho\xi\sigma_z^2\left[1+\theta\left(\frac{\eta_2^2C_x^2}{4} + \frac{3}{4}\eta_2C_x^2\right)\right]$$
$$+ \xi\sigma_z^2\left(\frac{1-\rho}{n}\right)\left[1+\theta\left(\frac{\eta_2^2C_x^2}{4} + \frac{3}{4}\eta_2C_x^2\right)\right] + \beta^2\bar{Z}^2 + 2\eta_2(1+\beta)\beta\bar{Z}^2\theta\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right),$$

and

$$M_I[T_1(\eta_1,\eta_2)] = (1+\beta)^2\bar{Z}^2\theta\left(C_{z'}^2 + \frac{\eta_2^2C_x^2}{4} - \eta_2C_{xz'}\right) + \left(\frac{n-1}{n}\right)\rho_w\xi\sigma_z^2$$
$$+ \frac{\xi\sigma_z^2}{n}\left[1+\theta\left(\frac{\eta_2^2C_x^2}{4} + \frac{3}{4}\eta_2C_x^2\right)\right] + \beta^2\bar{Z}^2 + 2\eta_2(1+\beta)\beta\bar{Z}^2\theta\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right).$$

Remark 4. *It can be seen that the combination of $\beta = 0.00$, $\xi = 0.00$, and $\rho = 0.00$ denotes the absence of ME. As a result, in terms of β and ξ, the expressions of $B_C[T_2(\alpha)]$, $B_I[T_2(\alpha)]$, $M_C[T_2(\alpha)]$ and $M_I[T_2(\alpha)]$ of the proposed estimator $T_2(\alpha)$ become*

$$B_C[T_2(\alpha)] = \beta\bar{Z} + (1-\alpha)(1+\beta)\bar{Z}\theta\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right) = B_I[T_2(\alpha)],$$

$$M_C[T_2(\alpha)] = (1+\beta)^2\bar{Z}^2\theta\left(C_{z'}^2 + \frac{(1-\alpha)^2C_x^2}{4} - (1-\alpha)C_{xz'}\right)$$
$$+ \rho\xi\sigma_z^2\left[1+\theta\left(\frac{(1-\alpha)^2C_x^2}{4} + 2(1-\alpha)\frac{3}{8}C_x^2\right)\right]$$
$$+ \xi\sigma_z^2\left(\frac{1-\rho}{n}\right)\left[1+\theta\left(\frac{(1-\alpha)^2C_x^2}{4} + 2(1-\alpha)\frac{3}{8}C_x^2\right)\right] + \beta^2\bar{Z}^2$$
$$+ 2(1-\alpha)(1+\beta)\beta\bar{Z}^2\theta\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right),$$

and

$$M_I[T_2(\alpha)] = (1+\beta)^2\bar{Z}^2\theta\left(C_{z'}^2 + \frac{(1-\alpha)^2C_x^2}{4} - (1-\alpha)C_{xz'}\right) + \left(\frac{n-1}{n}\right)\rho_w\xi\sigma_z^2$$
$$+ \frac{\xi\sigma_z^2}{n}\left[1+\theta\left(\frac{(1-\alpha)^2C_x^2}{4} + 2(1-\alpha)\frac{3}{8}C_x^2\right)\right] + \beta^2\bar{Z}^2$$
$$+ 2(1-\alpha)(1+\beta)\beta\bar{Z}^2\theta\left(\frac{3}{8}C_x^2 - \frac{C_{xz'}}{2}\right).$$

The optimum value of α_0 can be obtained as

$$[\hat{\alpha}_0]_C = \frac{(1+\beta)^2\bar{Z}^2\left(\frac{C_x^2}{2}-C_{xz'}\right)+\frac{5}{4}C_x^2\left(\frac{1-\rho}{n}\xi\sigma_z^2+\rho\xi\sigma_z^2\right)+2(1+\beta)\beta\bar{Z}^2\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)}{\frac{C_x^2}{2}\left((1+\beta)^2\bar{Z}^2+\left(\frac{1-\rho}{n}\right)\xi\sigma_z^2+\rho\xi\sigma_z^2\right)},$$

and under the ICM model, the optimum value is

$$[\hat{\alpha}_0]_I = \frac{(1+\beta)^2\bar{Z}^2\left(\frac{C_x^2}{2}-C_{xz'}\right)+\frac{5\xi\sigma_z^2}{4n}C_x^2+2(1+\beta)\beta\bar{Z}^2\left(\frac{3}{8}C_x^2-\frac{C_{xz'}}{2}\right)}{\frac{C_x^2}{2}\left((1+\beta)^2\bar{Z}^2+\frac{\xi\sigma_z^2}{n}\right)}.$$

10 Empirical study

We used the population from Kish (1965) for the empirical investigation. The information in Kish (1965) relates to a group of 270 blocks in Fall River, Ward I of Massachusetts, found in the Block Statistics volume of the 1950 U.S. Census. There are twenty-seven groupings of ten blocks each. The block numbers are 232 to 772. Out of the 541 block numbers, 271 are missing, indicating that the block was either in a different ward or had no residences. In the data, variable X indicates the total number of homes, whereas variable Z indicates the number of homes occupied by renters.

Since only 223 pairs of (X_i, Z_i) values out of 270 pairings have complete information for (X, Z) values, we take $N = 223$ into consideration here. The following population values are obtained by using the number of houses occupied by renters as the study variable Z and the number of dwellings as the auxiliary variable X, respectively:

$$N = 223, \quad \bar{Z} = 20.44, \quad \bar{X} = 29.93, \quad C_z^2 = 1.07, \quad C_x^2 = 0.70, \quad C_{xz} = 0.83 \quad \text{and} \quad \rho_{xz} = 0.9602.$$

For simplicity, we assume that the data on the variable Z are the actual values of Z, regardless of how the data are collected in the survey. The detailed results of the empirical study are given in Tables 1–6. These tables show the MSE values for the different sampling strategies for different sample sizes n, measurement-error levels ξ, values of β, and correlation ρ. To provide a concise visual summary of the main findings, Figures 1 to 4 present the corresponding results graphically, and the remaining Figures 5–10, which provide additional details of the numerical results.

The main findings from Figures 1 to 4 show how changes in the model parameters affect the performance of the different strategies:

(i) In Figure 1, we have $\beta = \xi = 0$ and $\rho = 0$; this means that there is absolutely no ME present in the data. The values in each table are identical, as expected. The method $[P, \bar{z}_{OR}]$ appears to perform better than $[P, T_e[1]]$, but this is because the requirement $C_{xz} < 3/4$, which states that an exponential-type ratio estimator is superior to a classical ratio estimator, is not met for the data set. It is clear that strategy $[P, T_e[1]]$ would be superior to $[P, \bar{z}_{OR}]$ even in the presence of ME if the condition were met. Despite this, the strategy $[P, T_2[\alpha]]$ approach is better than $[P, \bar{z}_{OR}]$ and $[P, T_e[\alpha]]$. Additionally, whatever the model type chosen, it remains superior to all other strategies.

(ii) Figures 1 to 4 show that as the sample size increases from 25 to 75, the MSE decreases, meaning the estimates become more accurate. The proposed estimator performs competitively and is generally efficient with the least error, while the traditional estimator shows the highest error in all cases.

(iii) When there is no measurement error, both the proposed and existing estimators perform at their best.

(iv) It has been found that, whatever the model selected or the existence of additional ME components, strategies $[P, T_2[\alpha]]$ and $[P, T_e[\alpha]]$ are consistently more stable for under-reporting values than for over-reporting values.

(v) As the value of ξ decreases, the efficiency of the estimators increases across all the parameters.

(vi) A substantial reduction in β leads to improved performance of the estimators across all parameters.

(vii) As shown in Table 1 and Table 2, increasing the sample size leads to a decrease in MSE, as expected.

(viii) It is evident that within-trial variability and the strong correlation between error terms are more significant than measurement error (response bias).

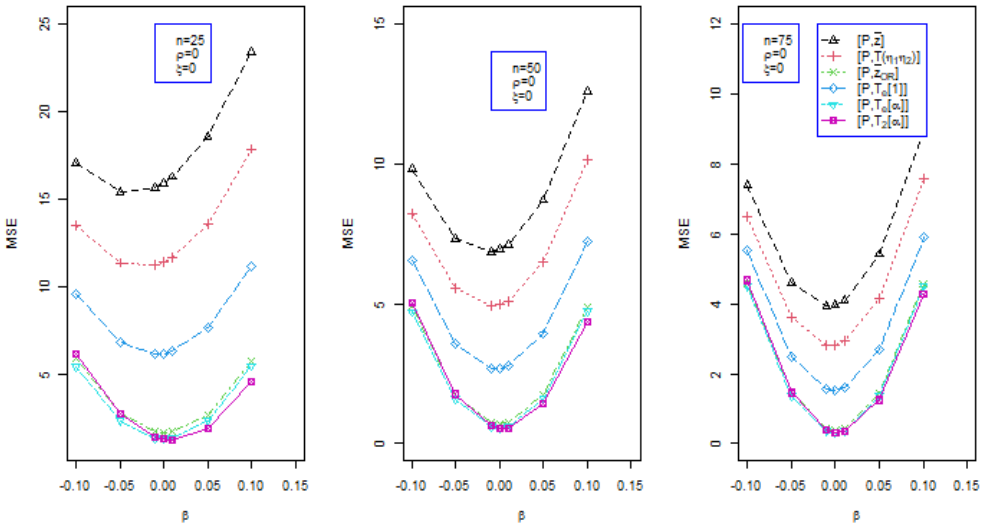


Figure 1: MSE of the sampling strategies under varying values of $\rho = 0$, $\xi = 0$, and n.

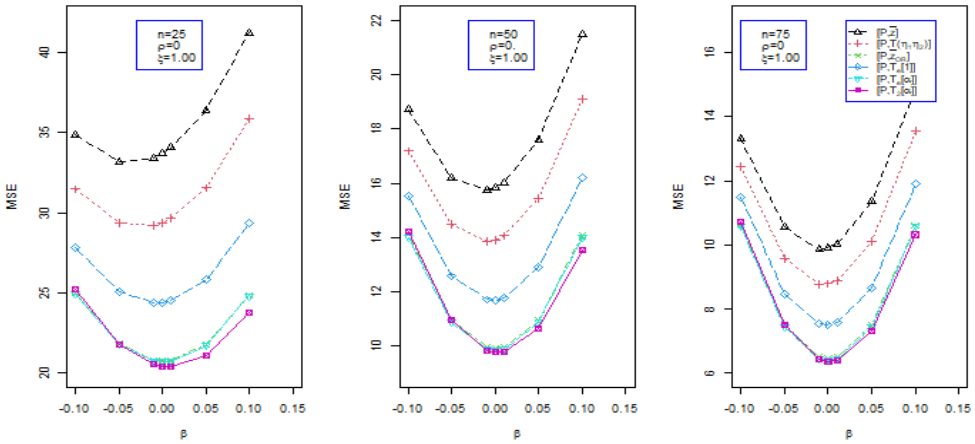


Figure 2: MSE of the sampling strategies under varying values of $\rho = 0$, $\xi = 1$, and n.

Based on these findings, it can be said that while ME is undoubtedly an important kind of error that has a significant impact on estimation processes, of its various components, greater variability in ME over time is more significant than response bias, whether it be under- or over-reporting. Therefore, the greatest caution is advised when measuring the observations at several trials. However, using professional interviewers should also minimise the participant's personal bias. On the other hand, as the study suggests, within-trial variability has a greater impact on the outcomes than response bias. The exponential-type estimators also perform well under different conditions and may be useful alternatives to commonly used estimators when measurement errors are present.

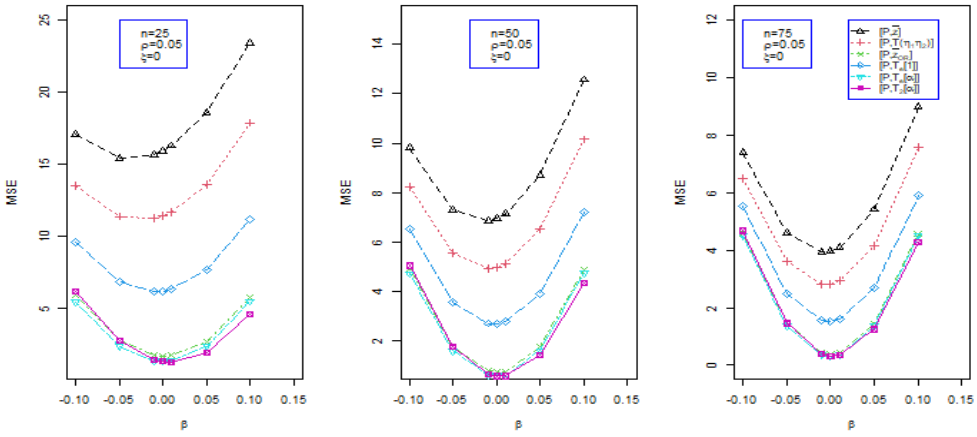


Figure 3: MSE of the sampling strategies under varying values of $\rho = 0.05$, $\xi = 0$, and n.

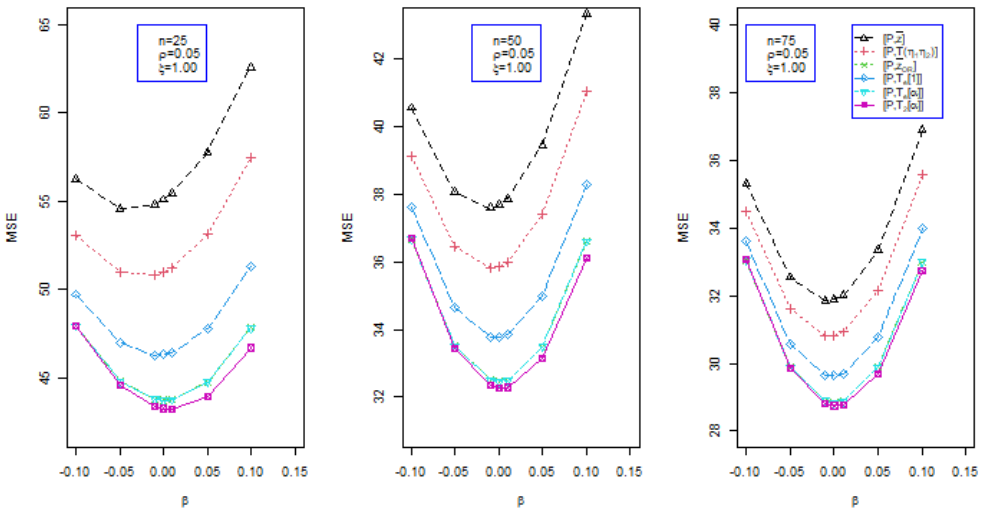


Figure 4: MSE of the sampling strategies under varying values of $\rho = 0.05$, $\xi = 1.00$, and n.

11 Conclusion

This study developed a class of estimators for estimating the population mean under measurement error. The results under the Constant Correlation Model (CCM) and the Inter-sample Correlation Model (ICM) show that the efficiency of the estimators is affected by the measurement-error parameters, the sample size, and the correlation structure. The numerical results show that the MSE generally decreases as the sample size increases, while it tends to increase with higher levels of measurement-error variability. Overall, the proposed estimators show competitive performance compared with commonly used estimators under a range of conditions. The empirical study also supports the usefulness of the proposed estimators for estimating the population mean under measurement error. Although the present study focuses on population mean estimation, the proposed estimators may also have potential applications in reliability and survival studies where lifetime or failure-time data are affected by measurement errors. This provides scope for future research on extending the proposed approach to more specific reliability and survival settings, including censored and incomplete lifetime data. Overall, the findings indicate that the proposed estimators can provide reliable estimates of the population mean under measurement error and may provide a useful basis for further work in reliability and survival applications.

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Data Availability

The data used in this study were taken from Kish (1965).

Declaration

The authors report no conflict of interest.

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Table 1: MSE of strategy with changing values of model parameters CCM (ρ=0).

ξ	n	β	$[P,\bar{z}]$	$[P,\bar{z}_{OR}]$	$[P,T_e[1]$	$[P,T_e[\alpha]$	$[P,T_1(\eta_1,\eta_2)]$	$[P,T_2[\alpha]$
0.00	25	-0.10	17.04	5.85	9.57	5.39	10.73	6.16
		-0.05	15.37	2.70	6.82	2.30	8.15	2.70
		-0.01	15.60	1.68	6.12	1.31	7.61	1.40
		0.00	15.88	1.63	6.16	1.27	7.69	1.27
		0.01	16.24	1.67	6.28	1.32	7.85	1.23
		0.05	18.55	2.64	7.60	2.32	9.33	1.88
		0.10	23.39	5.73	11.13	5.44	13.08	4.52
	50	-0.10	9.80	4.91	6.53	4.71	7.04	5.04
		-0.05	7.30	1.77	3.57	1.59	4.15	1.77
		-0.01	6.84	0.76	2.70	0.60	3.35	0.63
		0.00	6.94	0.71	2.69	0.56	3.36	0.56
		0.01	7.12	0.75	2.77	0.60	3.45	0.56
		0.05	8.69	1.74	3.91	1.60	4.66	1.41
		0.10	12.57	4.86	7.22	4.73	8.07	4.33
	75	-0.10	7.38	4.59	5.52	4.48	5.81	4.67
		-0.05	4.61	1.46	2.48	1.36	2.82	1.46
		-0.01	3.92	0.45	1.56	0.36	1.93	0.38
		0.00	3.96	0.41	1.53	0.32	1.92	0.32
		0.01	4.08	0.45	1.60	0.36	1.99	0.34
		0.05	5.41	1.44	2.68	1.36	3.11	1.25
		0.10	8.96	4.56	5.91	4.49	6.40	4.26
0.25	25	-0.10	21.49	10.63	14.14	10.31	15.27	10.94
		-0.05	19.83	7.49	11.38	7.20	12.69	7.49
		-0.01	20.06	6.47	10.69	6.20	12.15	6.20
		0.00	20.33	6.42	10.72	6.16	12.23	6.08
		0.01	20.69	6.46	10.84	6.20	12.39	6.04
		0.05	23.00	7.43	12.16	7.20	13.87	6.69
		0.10	27.84	10.52	15.70	10.31	17.62	9.34
	50	-0.10	12.02	7.21	8.79	7.04	9.29	7.34
		-0.05	9.53	4.07	5.82	3.92	6.40	4.07
		-0.01	9.07	3.06	4.95	2.92	5.59	2.94
		0.00	9.16	3.01	4.94	2.88	5.60	2.86
		0.01	9.34	3.05	5.02	2.92	5.70	2.87
		0.05	10.92	4.04	6.16	3.92	6.91	3.72
		0.10	14.80	7.16	9.47	7.05	10.31	6.63
	75	-0.10	8.87	6.11	7.02	6.00	7.30	6.18
		-0.05	6.10	2.97	3.98	2.88	4.31	2.97
		-0.01	5.40	1.96	3.05	1.88	3.42	1.89
		0.00	5.44	1.92	3.03	1.84	3.41	1.83
		0.01	5.56	1.96	3.09	1.88	3.48	1.85
		0.05	6.89	2.96	4.17	2.88	4.60	2.77
		0.10	10.45	6.08	7.40	6.01	7.89	5.78
0.50	25	-0.10	25.95	15.42	18.71	15.21	19.81	15.71
		-0.05	24.28	12.28	15.95	12.08	17.23	12.28
		-0.01	24.51	11.25	15.25	11.08	16.69	10.99
		0.00	24.79	11.21	15.29	11.03	16.76	10.87
		0.01	25.15	11.24	15.41	11.07	16.92	10.84
		0.05	27.46	12.22	16.73	12.06	18.41	11.50
		0.10	32.30	15.30	20.26	15.16	22.16	14.15
	50	-0.10	14.25	9.51	11.04	9.37	11.53	9.64
		-0.05	11.76	6.37	8.07	6.24	8.64	6.37
		-0.01	11.30	5.36	7.20	5.24	7.84	5.24
		0.00	11.39	5.31	7.19	5.20	7.85	5.16

Table 2: Continuation of Table 1.

ξ	n	β	$[P,\bar{z}]$	$[P,\bar{z}_{OR}]$	$[P,T_e[1]$	$[P,T_e[\alpha]$	$[P,T_1(\eta_1,\eta_2)]$	$[P,T_2[\alpha]$
0.5	50	0.01	11.57	5.35	7.27	5.24	7.94	5.17
		0.05	13.15	6.34	8.41	6.24	9.16	6.02
		0.10	17.03	9.46	11.72	9.37	12.56	8.94
0.5	75	-0.10	10.35	7.62	8.51	7.53	8.79	7.70
		-0.05	7.58	4.48	5.47	4.40	5.80	4.48
		-0.01	6.89	3.48	4.55	3.40	4.91	3.41
		0.00	6.93	3.43	4.52	3.36	4.90	3.35
		0.01	7.05	3.47	4.58	3.40	4.97	3.37
		0.05	8.38	4.47	5.67	4.40	6.09	4.28
		0.10	11.93	7.59	8.90	7.53	9.38	7.29
0.75	25	-0.10	30.40	20.21	23.27	20.08	24.35	20.47
		-0.05	28.74	17.06	20.51	16.94	21.77	17.05
		-0.01	28.97	16.04	19.82	15.93	21.23	15.78
		0.00	29.24	15.99	19.86	15.89	21.30	15.66
		0.01	29.60	16.03	19.98	15.92	21.46	15.62
		0.05	31.91	17.00	21.29	16.90	22.95	16.30
		0.10	36.75	20.09	24.83	20.00	26.70	18.96
	50	-0.10	16.48	11.81	13.29	11.69	13.78	11.94
		-0.05	13.99	8.67	10.32	8.56	10.89	8.67
		-0.01	13.52	7.66	9.45	7.56	10.09	7.54
		0.00	13.62	7.61	9.45	7.52	10.10	7.47
		0.01	13.80	7.65	9.52	7.56	10.19	7.47
		0.05	15.37	8.64	10.66	8.56	11.40	8.33
		0.10	19.25	11.76	13.97	11.68	14.80	11.25
	75	-0.10	11.84	9.13	10.00	9.05	10.29	9.21
		-0.05	9.07	6.00	6.97	5.92	7.29	6.00
		-0.01	8.37	4.99	6.04	4.92	6.40	4.92
		0.00	8.41	4.94	6.02	4.88	6.39	4.86
		0.01	8.53	4.98	6.08	4.92	6.46	4.88
		0.05	9.86	5.98	7.16	5.92	7.58	5.80
		0.10	13.42	9.10	10.39	9.05	10.87	8.81
1.00	25	-0.10	34.86	25.00	27.84	24.93	28.89	25.23
		-0.05	33.19	21.85	25.08	21.79	26.31	21.82
		-0.01	33.42	20.83	24.39	20.77	25.77	20.55
		0.00	33.70	20.78	24.42	20.72	25.84	20.44
		0.01	34.06	20.82	24.54	20.76	26.00	20.40
		0.05	36.37	21.79	25.86	21.74	27.49	21.08
		0.10	41.21	24.88	29.39	24.83	31.24	23.76
	50	-0.10	18.71	14.11	15.54	14.01	16.02	14.23
		-0.05	16.21	10.97	12.57	10.88	13.13	10.97
		-0.01	15.75	9.96	11.71	9.88	12.33	9.84
		0.00	15.85	9.91	11.70	9.84	12.34	9.77
		0.01	16.03	9.95	11.77	9.88	12.44	9.77
		0.05	17.60	10.94	12.91	10.87	13.65	10.63
		0.10	21.48	14.06	16.22	13.99	17.05	13.55
	75	-0.10	13.32	10.64	11.50	10.57	11.78	10.72
		-0.05	10.55	7.51	8.46	7.44	8.78	7.51
		-0.01	9.86	6.50	7.53	6.44	7.90	6.43
		0.00	9.90	6.46	7.51	6.40	7.88	6.37
		0.01	10.02	6.50	7.57	6.44	7.95	6.39
		0.05	11.35	7.49	8.65	7.44	9.08	7.31
		0.10	14.90	10.61	11.89	10.57	12.36	10.32

Table 3: MSE of strategy with changing values of model parameters CCM (ρ=0.05).

ξ	n	β	$[P,\bar{z}]$	$[P,\bar{z}_{OR}]$	$[P,T_e[1]$	$[P,T_e[\alpha]$	$[P,T_1(\eta_1,\eta_2)]$	$[P,T_2[\alpha]$
0.25	25	-0.10	26.84	16.38	19.62	16.18	20.72	16.66
		-0.05	25.17	13.23	16.86	13.06	18.14	13.23
		-0.01	25.40	12.21	16.17	12.05	17.60	11.95
		0.00	25.68	12.16	16.20	12.00	17.67	11.83
		0.01	26.04	12.20	16.32	12.04	17.83	11.79
		0.05	28.35	13.17	17.64	13.03	19.32	12.46
		0.10	33.19	16.26	21.18	16.13	23.07	15.12
	50	-0.10	17.48	12.84	14.30	12.74	14.79	12.97
		-0.05	14.99	9.70	11.33	9.61	11.90	9.70
		-0.01	14.53	8.69	10.47	8.61	11.10	8.58
		0.00	14.62	8.65	10.46	8.56	11.11	8.50
		0.01	14.80	8.69	10.53	8.60	11.20	8.51
		0.05	16.38	9.68	11.68	9.60	12.41	9.36
		0.10	20.26	12.79	14.98	12.72	15.82	12.29
	75	-0.10	14.36	11.70	12.55	11.64	12.82	11.78
		-0.05	11.59	8.57	9.51	8.51	9.83	8.57
		-0.01	10.90	7.56	8.58	7.51	8.94	7.49
		0.00	10.94	7.52	8.56	7.46	8.93	7.43
		0.01	11.06	7.56	8.62	7.51	9.00	7.45
		0.05	12.39	8.55	9.70	8.51	10.12	8.37
0.50	25	-0.10	36.64	26.91	29.66	26.86	30.70	27.13
		-0.05	34.98	23.77	26.91	23.72	28.13	23.72
		-0.01	35.21	22.74	26.21	22.70	27.58	22.46
		0.00	35.48	22.70	26.25	22.65	27.66	22.35
		0.01	35.84	22.73	26.37	22.69	27.82	22.32
		0.05	38.15	23.71	27.69	23.66	29.30	23.00
		0.10	42.99	26.79	31.22	26.75	33.05	25.68
	50	-0.10	25.17	20.78	22.07	20.74	22.54	20.89
		-0.05	22.67	17.64	19.10	17.60	19.65	17.63
		-0.01	22.21	16.63	18.24	16.59	18.85	16.51
		0.00	22.31	16.58	18.23	16.55	18.86	16.44
		0.01	22.49	16.62	18.30	16.59	18.95	16.45
		0.05	24.06	17.61	19.44	17.58	20.16	17.30
		0.10	27.94	20.73	22.75	20.70	23.56	20.24
	75	-0.10	21.34	18.81	19.57	18.78	19.84	18.88
		-0.05	18.57	15.68	16.53	15.65	16.84	15.67
		-0.01	17.88	14.67	15.60	14.64	15.95	14.60
		0.00	17.92	14.63	15.58	14.60	15.94	14.54
		0.01	18.04	14.67	15.64	14.64	16.01	14.56
		0.05	19.37	15.66	16.72	15.64	17.13	15.49
		0.10	22.92	18.78	19.96	18.76	20.42	18.50

Note: For $\xi = 0.00$ and $n = 25, 50$ and 75, the values are identical to the corresponding values reported in Table 1.

Table 4: Continuation of Table 3.

ξ	n	β	$[P,\bar{z}]$	$[P,\bar{z}_{OR}]$	$[P,T_e[1]$	$[P,T_e[\alpha]$	$[P,T_1(\eta_1,\eta_2)]$	$[P,T_2[\alpha]$
0.75	25	-0.10	46.44	37.44	39.71	37.44	40.69	37.54
		-0.05	44.78	34.30	36.95	34.30	38.11	34.17
		-0.01	45.01	33.28	36.26	33.28	37.57	32.93
		0.00	45.28	33.23	36.29	33.23	37.65	32.83
		0.01	45.64	33.27	36.41	33.26	37.81	32.80
		0.05	47.95	34.24	37.73	34.24	39.29	33.50
		0.10	52.79	37.33	41.27	37.32	43.04	36.21
	50	-0.10	32.85	28.71	29.84	28.71	30.29	28.79
		-0.05	30.36	25.57	26.87	25.57	27.40	25.55
		-0.01	29.90	24.56	26.00	24.55	26.59	24.43
		0.00	29.99	24.52	26.00	24.51	26.60	24.36
		0.01	30.17	24.56	26.07	24.55	26.70	24.37
		0.05	31.75	25.55	27.21	25.54	27.91	25.24
		0.10	35.63	28.66	30.52	28.65	31.31	28.18
	75	-0.10	28.32	25.92	26.59	25.91	26.85	25.97
		-0.05	25.55	22.79	23.55	22.78	23.85	22.77
		-0.01	24.86	21.78	22.63	21.77	22.97	21.71
		0.00	24.90	21.74	22.60	21.73	22.95	21.65
		0.01	25.02	21.78	22.66	21.77	23.02	21.67
		0.05	26.35	22.77	23.75	22.76	24.15	22.59
		0.10	29.90	25.89	26.98	25.89	27.43	25.62
1.00	25	-0.10	56.24	47.98	49.75	47.94	50.68	47.92
		-0.05	54.58	44.83	47.00	44.80	48.10	44.59
		-0.01	54.81	43.81	46.30	43.79	47.56	43.37
		0.00	55.08	43.76	46.34	43.74	47.63	43.27
		0.01	55.44	43.80	46.46	43.78	47.79	43.25
		0.05	57.75	44.77	47.78	44.75	49.28	43.98
		0.10	62.59	47.86	51.31	47.84	53.03	46.71
	50	-0.10	40.54	36.65	37.61	36.65	38.03	36.69
		-0.05	38.05	33.51	34.64	33.51	35.14	33.45
		-0.01	37.58	32.50	33.77	32.50	34.34	32.35
		0.00	37.68	32.46	33.76	32.46	34.35	32.27
		0.01	37.86	32.49	33.84	32.49	34.45	32.29
		0.05	39.43	33.48	34.98	33.48	35.66	33.16
		0.10	43.31	36.60	38.29	36.60	39.06	36.11
	75	-0.10	35.30	33.03	33.61	33.03	33.86	33.06
		-0.05	32.53	29.90	30.57	29.89	30.87	29.87
		-0.01	31.84	28.89	29.65	28.89	29.98	28.81
		0.00	31.88	28.84	29.63	28.84	29.97	28.75
		0.01	32.00	28.89	29.69	28.88	30.04	28.77
		0.05	33.33	29.88	30.77	29.88	31.16	29.70
		0.10	36.88	33.00	34.00	33.00	34.45	32.72

Table 5: MSE of strategy with changing values of model parameters ICM (ρ=0.05).

ξ	n	β	$[P,\bar{z}]$	$[P,\bar{z}_{OR}]$	$[P,T_e[1]$	$[P,T_e[\alpha]$	$[P,T_1(\eta_1,\eta_2)]$	$[P,T_2[\alpha]$
0.25	25	-0.10	26.84	15.98	19.49	15.66	20.62	16.29
		-0.05	25.17	12.83	16.73	12.55	18.04	12.84
		-0.01	25.40	11.81	16.03	11.55	17.50	11.55
		0.00	25.68	11.77	16.07	11.51	17.57	11.43
		0.01	26.04	11.80	16.19	11.55	17.73	11.39
		0.05	28.35	12.78	17.51	12.54	19.21	12.04
		0.10	33.19	15.86	21.04	15.66	22.97	14.69
	50	-0.10	17.48	12.67	14.24	12.50	14.74	12.80
		-0.05	14.99	9.53	11.28	9.38	11.85	9.53
		-0.01	14.53	8.52	10.41	8.38	11.05	8.40
		0.00	14.62	8.47	10.40	8.34	11.06	8.32
		0.01	14.80	8.51	10.48	8.38	11.15	8.32
		0.05	16.38	9.50	11.62	9.38	12.37	9.17
		0.10	20.26	12.61	14.92	12.51	15.77	12.09
	75	-0.10	14.36	11.60	12.51	11.50	12.80	11.68
		-0.05	11.59	8.46	9.47	8.37	9.80	8.46
		-0.01	10.90	7.46	8.55	7.38	8.91	7.39
		0.00	10.94	7.41	8.52	7.33	8.90	7.33
		0.01	11.06	7.45	8.58	7.38	8.97	7.35
		0.05	12.39	8.45	9.67	8.38	10.10	8.26
		0.10	15.94	11.57	12.90	11.51	13.38	11.27
0.50	25	-0.10	36.64	26.11	29.40	25.90	30.50	26.41
		-0.05	34.98	22.97	26.64	22.78	27.92	22.98
		-0.01	35.21	21.95	25.95	21.77	27.38	21.69
		0.00	35.48	21.90	25.98	21.73	27.46	21.57
		0.01	35.84	21.94	26.10	21.76	27.62	21.53
		0.05	38.15	22.91	27.42	22.75	29.10	22.20
		0.10	42.99	26.00	30.96	25.85	32.85	24.85
	50	-0.10	25.17	20.42	21.95	20.28	22.45	20.56
		-0.05	22.67	17.28	18.99	17.16	19.56	17.29
		-0.01	22.21	16.27	18.12	16.16	18.76	16.16
		0.00	22.31	16.23	18.11	16.12	18.77	16.08
		0.01	22.49	16.27	18.18	16.16	18.86	16.09
		0.05	24.06	17.26	19.33	17.16	20.07	16.94
	75	-0.10	21.34	18.61	19.50	18.52	19.78	18.69
		-0.05	18.57	15.47	16.46	15.39	16.79	15.47
		-0.01	17.88	14.46	15.53	14.39	15.90	14.40
		0.00	17.92	14.42	15.51	14.35	15.89	14.34
		0.01	18.04	14.46	15.57	14.39	15.96	14.36
		0.05	19.37	15.46	16.65	15.39	17.08	15.27

Note: For $\xi = 0.00$ and $n = 25, 50$ and 75, the values are identical to the corresponding values reported in Table 1.

Table 6: Continuation of Table 5.

ξ	n	β	$[P,\bar{z}]$	$[P,\bar{z}_{OR}]$	$[P,T_e[1]$	$[P,T_e[\alpha]$	$[P,T_1(\eta_1,\eta_2)]$	$[P,T_2[\alpha]$
0.75	25	-0.10	46.44	36.25	39.31	36.12	40.39	36.52
		-0.05	44.78	33.10	36.55	32.98	37.81	33.10
		-0.01	45.01	32.08	35.86	31.97	37.27	31.83
		0.00	45.28	32.03	35.89	31.92	37.34	31.71
		0.01	45.64	32.07	36.01	31.96	37.50	31.68
		0.05	47.95	33.04	37.33	32.94	38.99	32.35
		0.10	52.79	36.13	40.87	36.04	42.74	35.01
	50	-0.10	32.85	28.18	29.66	28.06	30.15	28.31
		-0.05	30.36	25.04	26.69	24.94	27.26	25.04
		-0.01	29.90	24.03	25.83	23.94	26.46	23.92
		0.00	29.99	23.99	25.82	23.89	26.47	23.84
		0.01	30.17	24.03	25.89	23.93	26.56	23.85
		0.05	31.75	25.02	27.03	24.93	27.77	24.70
		0.10	35.63	28.13	30.34	28.05	31.18	27.62
	75	-0.10	28.32	25.62	26.49	25.54	26.77	25.69
		-0.05	25.55	22.48	23.45	22.41	23.78	22.48
		-0.01	24.86	21.47	22.52	21.41	22.89	21.41
		0.00	24.90	21.43	22.50	21.36	22.88	21.34
		0.01	25.02	21.47	22.56	21.41	22.95	21.37
		0.05	26.35	22.46	23.64	22.41	24.07	22.28
		0.10	29.90	25.59	26.88	25.53	27.36	25.29
1.00	25	-0.10	56.24	46.38	49.22	46.32	50.27	46.63
		-0.05	54.58	43.24	46.47	43.17	47.70	43.22
		-0.01	54.81	42.21	45.77	42.15	47.15	41.96
		0.00	55.08	42.17	45.81	42.11	47.23	41.84
		0.01	55.44	42.20	45.93	42.14	47.39	41.81
		0.05	57.75	43.18	47.24	43.12	48.87	42.49
		0.10	62.59	46.26	50.78	46.21	52.62	45.17
	50	-0.10	40.54	35.94	37.37	35.84	37.85	36.07
		-0.05	38.05	32.80	34.40	32.71	34.96	32.80
		-0.01	37.58	31.79	33.54	31.71	34.16	31.68
		0.00	37.68	31.74	33.53	31.67	34.17	31.60
		0.01	37.86	31.78	33.60	31.71	34.27	31.61
		0.05	39.43	32.77	34.74	32.70	35.48	32.46
		0.10	43.31	35.89	38.05	35.82	38.88	35.39
	75	-0.10	35.30	32.62	33.48	32.55	33.76	32.70
		-0.05	32.53	29.49	30.44	29.42	30.76	29.49
		-0.01	31.84	28.48	29.51	28.42	29.87	28.41
		0.00	31.88	28.44	29.49	28.38	29.86	28.35
		0.01	32.00	28.48	29.55	28.42	29.93	28.37
		0.05	33.33	29.47	30.63	29.42	31.06	29.29
		0.10	36.88	32.59	33.87	32.55	34.34	32.30

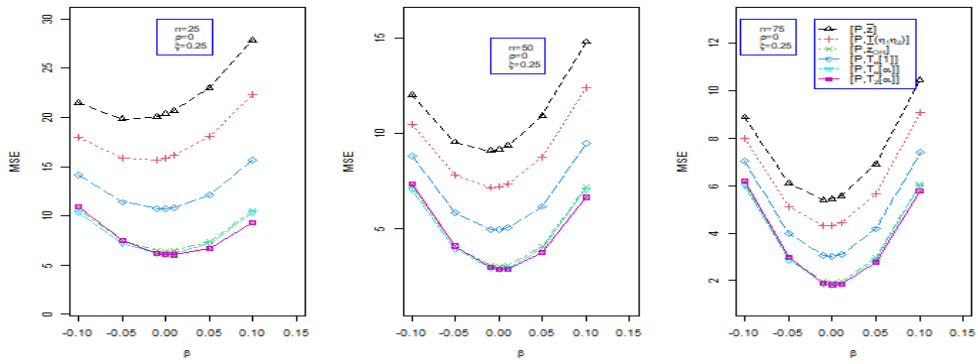


Figure 5: MSE of the sampling strategies under varying values of $\rho = 0$, $\xi = 0.25$, and n.

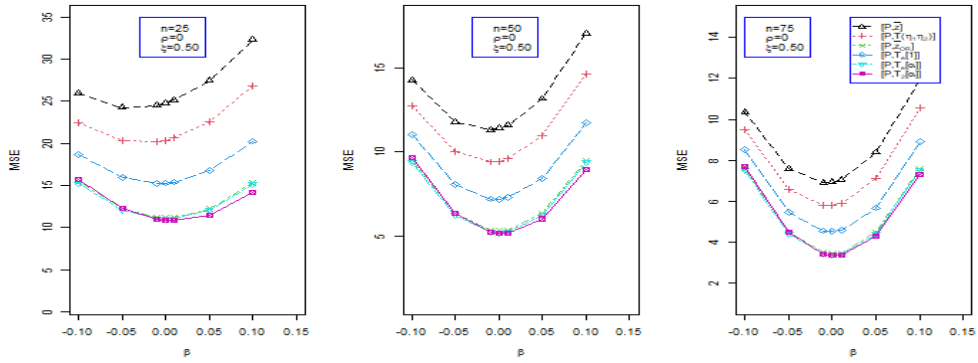


Figure 6: MSE of the sampling strategies under varying values of $\rho = 0$, $\xi = 0.50$, and n.

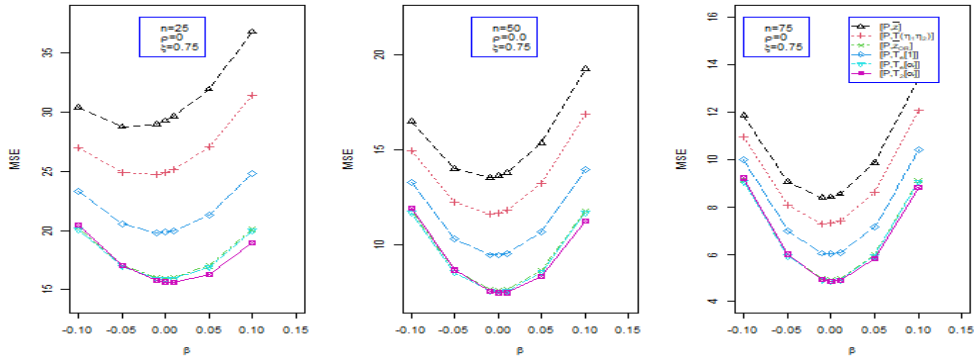


Figure 7: MSE of the sampling strategies under varying values of $\rho = 0$, $\xi = 0.75$, and n.

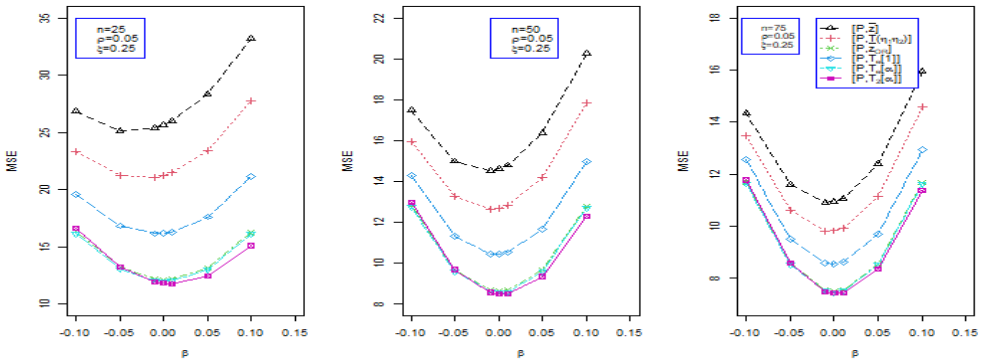


Figure 8: MSE of the sampling strategies under varying values of $\rho = 0.05$, $\xi = 0.25$, and n.

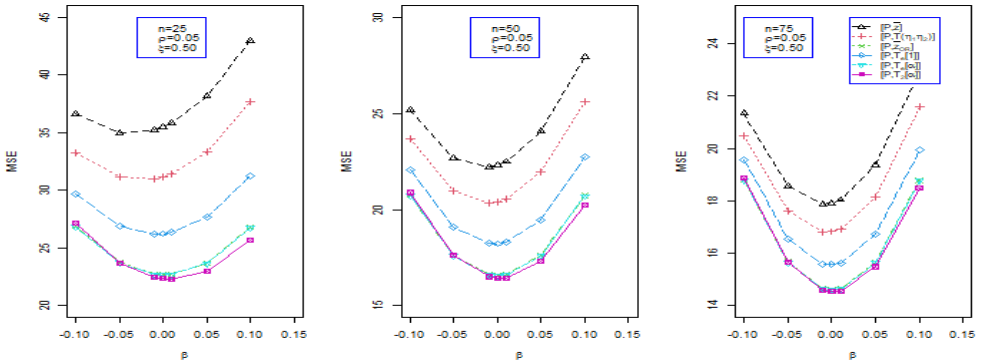


Figure 9: MSE of the sampling strategies under varying values of $\rho = 0.05$, $\xi = 0.50$, and n.

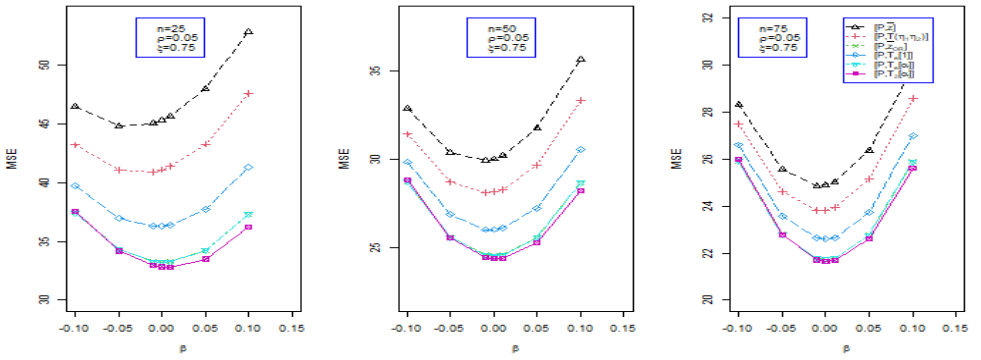


Figure 10: MSE of the sampling strategies under varying values of $\rho = 0.05$, $\xi = 0.75$, and n.